

Reliability of Supplemental Irrigation Supply under Dependent Precipitation and Evaporation: A Copula-Based FORM Analysis for Grassland in the Netherlands

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Abstract: Dry, sunny months combine low precipitation with high evaporative demand, so irrigation requirements depend on the joint behaviour of precipitation (P) and reference evaporation (E). This study evaluated the reliability of supplemental irrigation supply for grassland at three Dutch sites (De Bilt, 1980–2019; Heibloem, 1980–2016; Wageningen, 1976–1999) using monthly P and Makkink E. The net irrigation requirement (NIR) was computed for the growing season (April–September) and for the peak month (July). P and E were modelled with marginal distributions selected by the Anderson–Darling statistic, and their dependence with Gaussian, Frank and rotated Clayton and Gumbel copulas. The limit state compared a supply capacity with the NIR, which also depended on a crop factor, an effective rainfall fraction and the initial soil water. The first-order reliability method (FORM) with a Rosenblatt transformation was compared with the second-order method (SORM), Monte Carlo simulation (MCS) and importance sampling. Kendall's τ between P and E was -0.26 to -0.17 for the season and -0.51 to -0.35 for July. Seasonal E increased significantly at all sites. For a capacity equal to the empirical 80% quantile of the NIR, the copula model gave failure probabilities of 0.197–0.244. Ignoring dependence underestimated them by 8–10% for the season and 32–43% for July. FORM errors ranged from -8.5% to +10.2% relative to MCS, whereas SORM was within 1.3%. The order of the Rosenblatt transformation changed the FORM importance factors considerably but the failure probability only slightly. Inverse FORM showed that neglecting dependence would undersize the capacity for 95% reliability by 3–11% (season) and 9–17% (July). Precipitation was the most important variable, followed by E and the crop factor. Copula-based reliability analysis with SORM or MCS verification is recommended for sizing irrigation supplies.

Keywords: Irrigation requirement, Reliability, FORM, Copula, Rosenblatt transformation, Makkink evaporation

1. Introduction

Although the Netherlands has a humid climate, summer droughts such as those of 1976, 2003 and 2018 caused large agricultural losses and increased the demand for supplemental irrigation from groundwater and surface water. Drought trends in the Netherlands are driven by both precipitation and increasing potential evaporation, and they differ between regions (Philip et al., 2020). Irrigation systems, pumping permits and storage must therefore be sized for a demand that varies strongly between years.

The net irrigation requirement (NIR) is the difference between crop water demand and the water supplied by effective rainfall and soil storage (Brouwer and Heibloem, 1986; Allen et al., 1998). Reference evaporation is estimated in the Netherlands with the Makkink equation (Makkink, 1957; de Bruin and Lablans, 1998), and continental-scale studies have mapped irrigation requirements from such data (Wriedt et al., 2009). Most design methods use a fixed percentile of the historical NIR, which ignores the uncertainty in crop and soil parameters and the joint probability of low precipitation and high evaporation. Dry periods are usually sunny and warm, so precipitation and

evaporative demand are negatively correlated. Soil moisture–climate feedbacks can strengthen this link during droughts (Seneviratne et al., 2010).

Structural reliability methods provide a consistent framework for such problems. The first-order reliability method (FORM) finds the most probable failure point in standard normal space (Hasofer and Lind, 1974; Rackwitz and Fiessler, 1978). Second-order methods and simulation can verify it (Breitung, 1984; Melchers and Beck, 2018). Dependent non-normal variables must first be transformed to independent standard normals, either by the Nataf transformation (Der Kiureghian and Liu, 1986) or by the Rosenblatt (1952) transformation. Copulas give a flexible description of dependence for this purpose (Nelsen, 2006; Genest and Favre, 2007), and the Rosenblatt transformation can be written with copula conditional distributions. Lebrun and Dutfoy (2009) analysed how the two transformations relate, and for non-Gaussian copulas the order of conditioning in the Rosenblatt transformation can affect FORM results. Inverse reliability methods further allow a design value to be found for a target reliability (Der Kiureghian et al., 1994; Tu et al., 1999).

The objective of this study was to quantify the reliability of supplemental irrigation supply for grassland at three Dutch sites using monthly precipitation and Makkink evaporation. The specific aims were to (i) characterise P , E and their dependence for the growing season and the peak month; (ii) select marginal distributions and copulas; (iii) compare FORM, SORM, importance sampling and Monte Carlo simulation, including the effects of the copula family and the Rosenblatt ordering; (iv) quantify how dependence changes failure probabilities and design capacities; and (v) identify the most important random variables.

2. Materials and Methods

2.1. Sites and Data

Three records were used (Figure 1; Table 1). For De Bilt (KNMI station 260), daily precipitation and Makkink reference evaporation for January 1980 to March 2020 were taken from the example data of the Pastas package (Collenteur et al., 2019). For Heibloem in the south-eastern Netherlands, the same package provides daily precipitation measured at Heibloem and Makkink evaporation from the Maastricht station for 1980–2016. For Wageningen (Haarweg), daily weather for 1976–1999 was taken from the reference weather files of the PCSE crop simulation package (de Wit et al., 2019). Makkink evaporation was computed from global radiation and temperature with the coefficient 0.65 (de Bruin and Lablans, 1998). Only complete months were used. Monthly totals were summed over April–September to give the growing-season values, and July was analysed separately as the peak-demand month. Complete seasons were available for 40 (De Bilt), 37 (Heibloem) and 23 (Wageningen) years; the Wageningen season of 1991 was incomplete.

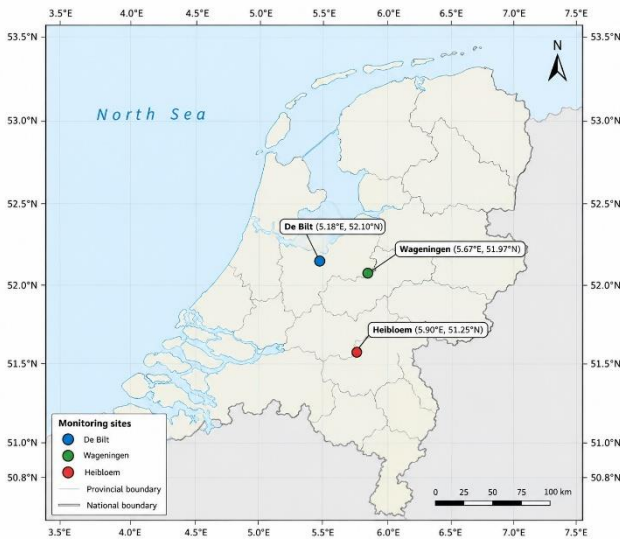


Figure 1. Approximate locations of the three sites in the Netherlands

Trends in seasonal and July P and E were tested with the Mann–Kendall test and the Sen slope.

2.2. Net Irrigation Requirement and Limit State

For grassland, the seasonal NIR (mm) was defined as

$$\text{NIR} = \max(0, K_c E - f_e P - SW_0) \quad (1)$$

where K_c is the crop factor relative to Makkink evaporation, f_e is the fraction of precipitation that is effective, and SW_0 is the plant-available soil water at the start of the season (Brouwer and Heibloem, 1986). For July, SW_0 was omitted because the soil reservoir is usually depleted by mid-summer. Irrigation losses were not included, so NIR is a net requirement. The supply fails when NIR exceeds the capacity W (mm per season or per month) that the system can deliver:

$$g(\mathbf{X}) = W - \text{NIR}(P, E, K_c, f_e, SW_0) \quad (2)$$

The five random variables are listed in Table 2. K_c was lognormal with a mean of 0.95 and a CoV of 0.08, which represents grassland whose crop factor relative to Makkink evaporation is close to one. f_e was beta-distributed with a mean of 0.80 and a standard deviation of 0.08, and SW_0 was lognormal with a mean of 50 mm and a CoV of 0.30, typical of sandy soils. These values are assumptions for a generic grassland on sandy soil and were kept the same at all sites. The empirical, or "nominal", NIR of each year was computed with the mean values of these three parameters.

2.3. Marginal Distributions and Copulas

Normal, gamma, lognormal and Weibull distributions were fitted to P and E by maximum likelihood, and the one with the smallest Anderson–Darling statistic was selected. Because P and E are negatively dependent, four copulas able to represent negative dependence were fitted by maximum pseudo-likelihood (Genest and Favre, 2007): the Gaussian and Frank copulas, and the Clayton and Gumbel copulas rotated by 90°:

$$C^{90}(u, v) = v - C(1 - u, v) \quad (3)$$

Goodness of fit was tested with the Cramér–von Mises statistic and a parametric bootstrap with 500 replicates (Genest et al., 2009). Among the copulas not rejected at the 5% level, the one with the smallest AIC was selected.

2.4. Reliability Methods

The variables were mapped to independent standard normal variables $\mathbf{U}$ with the Rosenblatt (1952) transformation, written in terms of the copula conditional distribution $h(v|u) = \partial C(u, v) / \partial u$:

$$u_p = F_P(P), \quad U_1 = \Phi^{-1}(u_p) \quad (4)$$

$$U_2 = \Phi^{-1}[h(F_E(E) | u_p)] \quad (5)$$

The other three variables were transformed independently. This ordering ($P \rightarrow E$) was compared with the reverse ordering ($E \rightarrow P$), which uses the conditional distribution of P given E . The reliability index β and design point were found with the HL-RF algorithm with a merit-function line search (Hasofer and Lind, 1974; Rackwitz and Fiessler, 1978), and $P_f \approx \Phi(-\beta)$. The importance factors are the squared components of the unit normal vector at the design point. The FORM

results were verified with the SORM approximations of Breitung (1984) and Hohenbichler and Rackwitz (1988), with crude MCS (2×10^6 samples) and with importance sampling (IS) centred on the design point (2×10^4 samples) (Melchers and Beck, 2018). The same analysis was repeated with the Frank copula and with independent P and E.

The reference capacity W_0 at each site was set to the empirical 80% quantile of the nominal NIR, a typical deterministic design. Failure probability curves were computed for a range of capacities. For design, the inverse FORM problem $\beta(W) = \beta_t$ was solved for target reliabilities of 0.80, 0.90 and 0.95 ($\beta_t = 0.84, 1.28$ and

1.64) (Der Kiureghian et al., 1994). Each design was checked by MCS with 10^6 samples. To isolate the effect of dependence, the failure probability at W_0 was also computed with a Gaussian copula whose Kendall's τ was 0, 0.5, 1 and 1.5 times the observed value. Finally, the model was refitted to the first and second halves of each record to examine changes in failure probability. All computations were coded in Python (NumPy, SciPy, pandas).

3. Results and Discussion

3.1. Precipitation, Evaporation and Irrigation Requirement

Table 1. Statistics of precipitation (P), Makkink evaporation (E) and nominal net irrigation requirement (NIR)

Site	Period	Years (n)	P (mm)	E (mm)	Kendall τ (p)	Sen slope E, mm yr ⁻¹ (p)	Sen slope P, mm yr ⁻¹ (p)	Mean NIR (mm)	NIR Q80 (mm)	Years NIR = 0 (%)	Max NIR year
De Bilt	Apr–Sep	1980–2019 (40)	405 ± 92	465 ± 36	-0.17 (0.121)	+2.00 (0.000)	+1.65 (0.196)	79	142	18	2018
	July	1980–2019 (40)	85 ± 49	97 ± 15	-0.51 (0.000)	+0.40 (0.059)	+0.74 (0.357)	35	69	32	2018
Heibloem	Apr–Sep	1980–2016 (37)	383 ± 75	476 ± 30	-0.19 (0.105)	+1.41 (0.001)	+0.91 (0.367)	102	163	11	2003
	July	1980–2016 (37)	73 ± 42	98 ± 15	-0.35 (0.002)	+0.23 (0.340)	+0.84 (0.147)	41	69	16	2006
Wageningen	Apr–Sep	1976–1999 (23)	350 ± 82	438 ± 37	-0.26 (0.091)	+3.13 (0.045)	+3.18 (0.267)	91	156	17	1976
	July	1976–1999 (24)	59 ± 38	91 ± 14	-0.36 (0.015)	+0.85 (0.087)	+0.04 (0.941)	45	67	17	1983

P and E as mean ± standard deviation; trends by the Mann–Kendall test; nominal NIR with $K_c = 0.95$, $f_e = 0.80$ and $SW_0 = 50$ mm (season only).

Mean growing-season precipitation was 350–405 mm and Makkink evaporation 438–477 mm, so the season had a climatic deficit (Table 1; Figure 2). The mean nominal NIR was 79–102 mm per season and 35–45 mm in July, and irrigation was not needed in 11–18% of the seasons. The largest seasonal requirements

occurred in 2018 at De Bilt, 2003 at Heibloem and 1976 at Wageningen (Figure 4), the known drought years. Seasonal E increased significantly at all three sites, by 1.4–3.1 mm per year. Seasonal P also tended to increase, but not significantly.

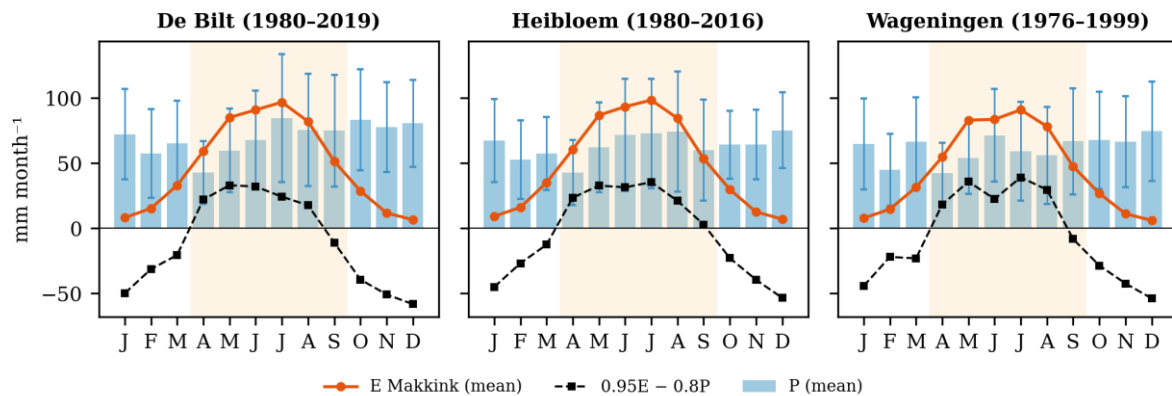


Figure 2. Mean monthly precipitation (± 1 SD), Makkink evaporation and nominal monthly deficit; the growing season is shaded

P and E were negatively dependent at all sites (Figure 3). The dependence was weak and not significant for the season ($\tau = -0.26$ to -0.17 ; $p = 0.09$ – 0.12). It was stronger and significant for July ($\tau = -0.51$ to -0.35 ; $p \leq 0.015$), when a dry month is also a sunny month. The seasonal

dependence was weakened by the simultaneous upward trends in P and E. Within the first and second halves of the record, τ for the season was -0.52 to -0.29 , stronger than for the whole record (Table 5).

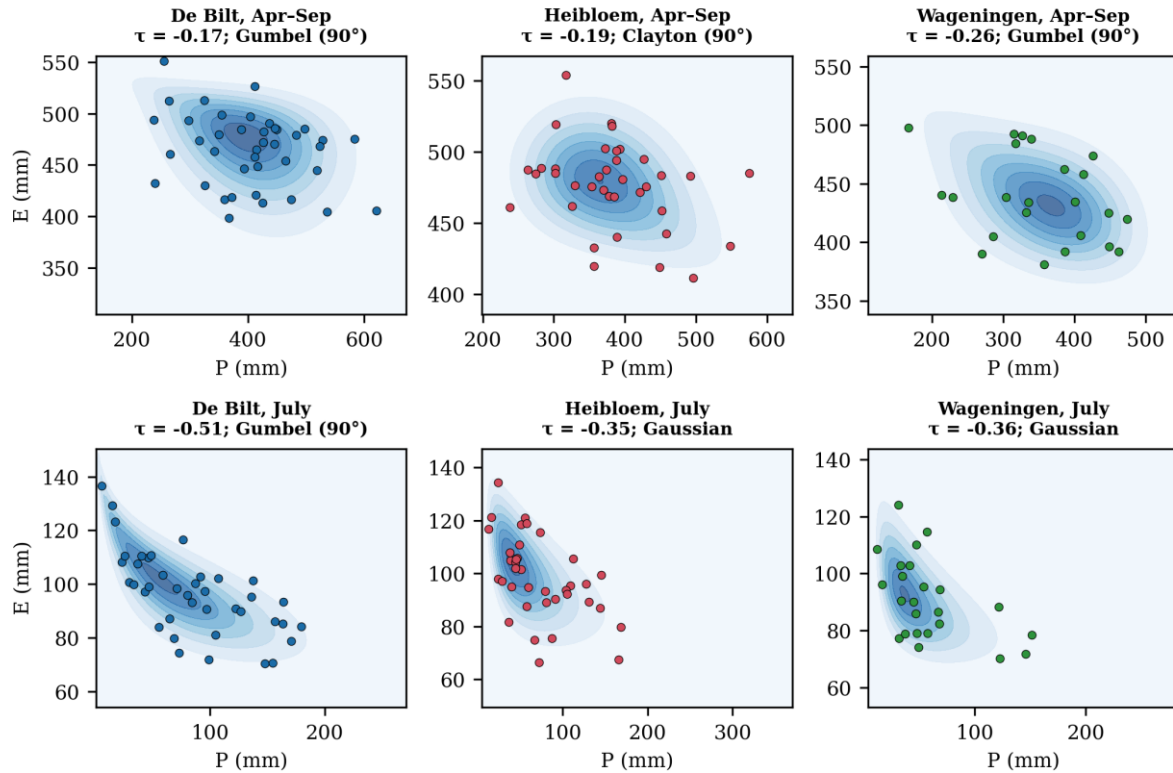


Figure 3. Seasonal and July precipitation versus Makkink evaporation with the joint density of the selected marginal–copula model

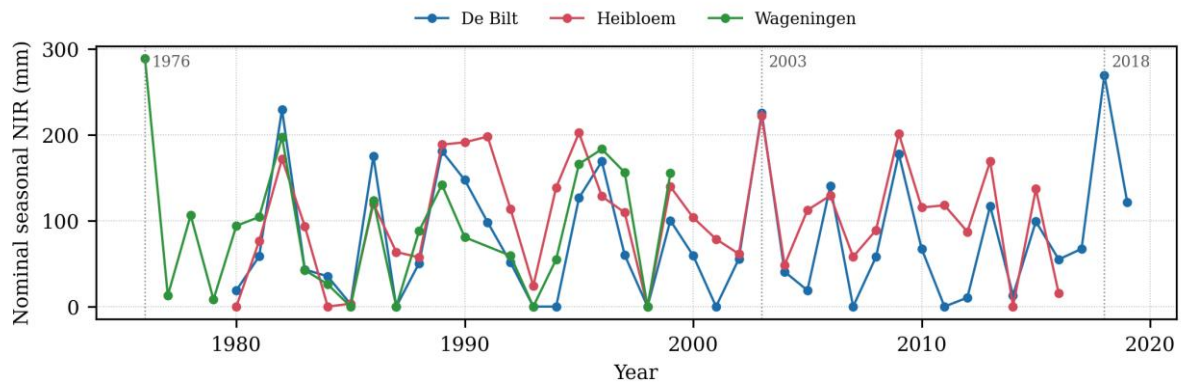


Figure 4. Nominal seasonal net irrigation requirement at the three sites

3.2. Marginal Distributions and Copulas

Table 2. Selected marginal distributions (KS p-value) and copula fits (AIC / Cramér–von Mises p-value)

Site	Period	P	E	Gaussian	Frank	Clayton 90°	Gumbel 90°	Selected copula
De Bilt	Apr–Sep	Normal (0.95)	Weibull (0.86)	-2.5 / 0.191	-0.8 / 0.109	-0.9 / 0.115	-2.8 / 0.350	Gumbel (90°) ($\theta = 1.29$)
	July	Weibull (0.97)	Gamma (0.98)	-24.6 / 0.807	-21.4 / 0.708	-11.4 / 0.053	-28.8 / 0.971	Gumbel (90°) ($\theta = 2.13$)
Heibloem	Apr–Sep	Gamma (0.80)	Normal (0.61)	-1.1 / 0.779	-0.7 / 0.759	-2.2 / 0.955	0.7 / 0.554	Clayton (90°) ($\theta = 0.55$)
	July	Lognormal (0.98)	Normal (0.99)	-11.9 / 0.037	-10.7 / 0.039	-7.8 / 0.025	-11.3 / 0.015	Gaussian ($\theta = -0.61$)
Wageningen	Apr–Sep	Weibull (0.96)	Lognormal (0.90)	-1.6 / 0.803	-1.0 / 0.783	-0.1 / 0.622	-2.2 / 0.843	Gumbel (90°) ($\theta = 1.40$)
	July	Lognormal (0.71)	Lognormal (0.73)	-6.5 / 0.781	-5.4 / 0.751	-6.1 / 0.544	-5.2 / 0.742	Gaussian ($\theta = -0.61$)

Random variables not shown: K_c lognormal (mean 0.95, CoV 0.08); f_e beta (mean 0.80, SD 0.08); SW_0 lognormal (mean 50 mm, CoV 0.30).

No single distribution fitted best everywhere (Table 2). Seasonal P was best described by normal, gamma or Weibull distributions, and July P by right-skewed Weibull or lognormal distributions. All selected

marginals passed the KS test ($p \geq 0.61$). The copula tests had limited power for 23–40 observations: all four copulas were accepted for the season at every site and for July at De Bilt and Wageningen. At Heibloem in

July, all four copulas were rejected at the 5% level ($p = 0.015\text{--}0.039$). The Gaussian copula, which had the smallest AIC, was used there, and results for that case should be treated with caution. The rotated Gumbel

copula was selected for three cases, and the Gaussian and rotated Clayton copulas for the others.

3.3. Comparison of Reliability Methods

Table 3. Failure probability at the reference capacity W_0 (empirical Q80 of NIR) by FORM, SORM-HR, IS and MCS

Site	Period	Dependence model	β (FORM)	P_f FORM	P_f SORM-HR	P_f IS	P_f MCS	FORM error (%)	SORM error (%)	$\alpha^2 P / E$
De Bilt	Apr-Sep	Selected, P→E	0.718	0.2365	0.2386	0.2377	0.2375	-0.4	+0.4	0.69 / 0.08
		Selected, E→P	0.751	0.2263	0.2404	0.2366	0.2373	-4.6	+1.3	0.38 / 0.38
		Frank, P→E	0.700	0.2420	0.2412	0.2393	0.2400	+0.9	+0.5	0.67 / 0.09
		Independent	0.770	0.2206	0.2214	0.2205	0.2200	+0.3	+0.6	0.59 / 0.14
	July	Selected, P→E	0.846	0.1988	0.1975	0.1969	0.1972	+0.8	+0.2	0.93 / 0.04
		Selected, E→P	0.864	0.1937	0.1972	0.1964	0.1974	-1.8	-0.1	0.84 / 0.12
		Frank, P→E	0.827	0.2041	0.2057	0.2046	0.2040	+0.1	+0.8	0.89 / 0.06
		Independent	1.063	0.1440	0.1396	0.1388	0.1383	+4.1	+1.0	0.72 / 0.21
	Apr-Sep	Selected, P→E	0.740	0.2297	0.2326	0.2347	0.2317	-0.9	+0.4	0.52 / 0.10
		Selected, E→P	0.717	0.2367	0.2321	0.2340	0.2314	+2.3	+0.3	0.25 / 0.38
		Frank, P→E	0.745	0.2283	0.2286	0.2312	0.2283	-0.0	+0.1	0.56 / 0.10
		Independent	0.813	0.2081	0.2123	0.2138	0.2113	-1.5	+0.4	0.46 / 0.14
Heibloem	July	Selected, P→E	0.676	0.2494	0.2438	0.2445	0.2437	+2.4	+0.1	0.81 / 0.12
		Selected, E→P	0.676	0.2494	0.2438	0.2430	0.2435	+2.5	+0.2	0.68 / 0.25
		Frank, P→E	0.649	0.2581	0.2516	0.2496	0.2485	+3.9	+1.2	0.82 / 0.11
		Independent	0.832	0.2028	0.1853	0.1856	0.1840	+10.2	+0.7	0.59 / 0.31
	Apr-Sep	Selected, P→E	0.802	0.2112	0.2218	0.2233	0.2224	-5.0	-0.3	0.72 / 0.09
		Selected, E→P	0.831	0.2031	0.2238	0.2219	0.2220	-8.5	+0.8	0.52 / 0.28
		Frank, P→E	0.778	0.2183	0.2282	0.2289	0.2282	-4.3	-0.0	0.69 / 0.11
		Independent	0.892	0.1862	0.2049	0.2042	0.2031	-8.3	+0.9	0.57 / 0.17
	July	Selected, P→E	0.689	0.2453	0.2417	0.2425	0.2414	+1.6	+0.1	0.77 / 0.15
		Selected, E→P	0.689	0.2453	0.2417	0.2405	0.2413	+1.7	+0.2	0.72 / 0.21
		Frank, P→E	0.663	0.2538	0.2480	0.2464	0.2453	+3.4	+1.1	0.78 / 0.14
		Independent	0.854	0.1966	0.1836	0.1841	0.1828	+7.6	+0.5	0.51 / 0.37

MCS with 2×10^6 samples ($\text{CoV} \leq 0.0018$); IS with 2×10^4 samples ($\text{CoV} \leq 0.009$); errors relative to MCS. $W_0 = 142, 69, 163, 69, 156, 67$ mm for the six cases in table order.

At the reference capacity, the MCS failure probability with the selected copula was 0.197–0.244 (Table 3), higher than the nominal 0.20 implied by the empirical 80% quantile (Figure 5). The difference comes from the uncertainty of the crop factor, effective rainfall and soil water, which the empirical quantile ignores. FORM required 66–99 limit-state evaluations and less than 0.2 s, compared with 3.5–9.7 s for MCS. Its errors relative to MCS ranged from -8.5% to +10.2%. The largest errors occurred for the independent case at Heibloem in July and for Wageningen in the season, where the $\max(0, \cdot)$ term and the skewed marginals make the limit state non-linear. The Breitung SORM reduced the errors to 4.1% and the Hohenbichler–

Rackwitz SORM to 1.3%. Importance sampling agreed with MCS within 1.3% at a hundredth of the sample size.

The ordering of the Rosenblatt transformation changed β by up to 0.033 and the FORM probability by up to 4%. The MCS probability, which is independent of the ordering, did not change. The importance factors changed much more (Figure 6). With the P→E ordering, P received most of the importance and E only a small share, because the effect of E is partly carried by P through the conditional transformation. With E→P, the importance was shared more evenly; at De Bilt in the season it was split almost equally (0.38 and 0.38). Importance factors of dependent variables thus depend on the chosen transformation (cf. Lebrun and Dutfoy, 2009) and should be interpreted with care.

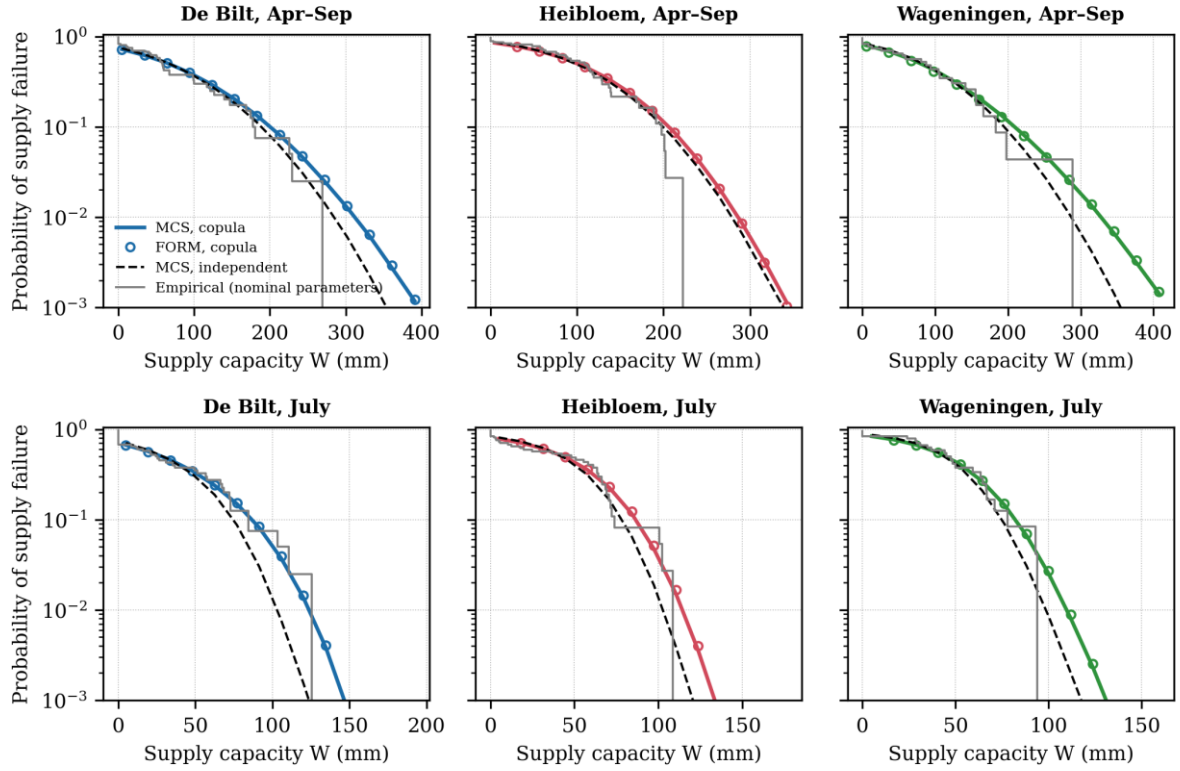


Figure 5. Probability of supply failure versus capacity: copula model (MCS line, FORM circles), independence (dashed) and the empirical exceedance frequency of the nominal NIR

3.4. Effect of Dependence

Negative dependence between P and E increased the failure probability at all capacities (Figure 5). At W_0 , ignoring dependence underestimated the MCS failure probability by 8–10% for the season and by 32–43% for July. The largest effect occurred at De Bilt in July (0.138 against 0.197), where the dependence was strongest. The relative underestimation grew with capacity, i.e. for the rare, high-requirement years that govern design. With

the Gaussian copula and scaled τ , the failure probability increased monotonically with the strength of dependence (Figure 7). At De Bilt in July it rose from 0.138 ($\tau = 0$) to 0.211 (1.5 times the observed τ). The copula family mattered less than the presence of dependence: the Frank copula changed the failure probability by at most 3.5% relative to the selected copula.

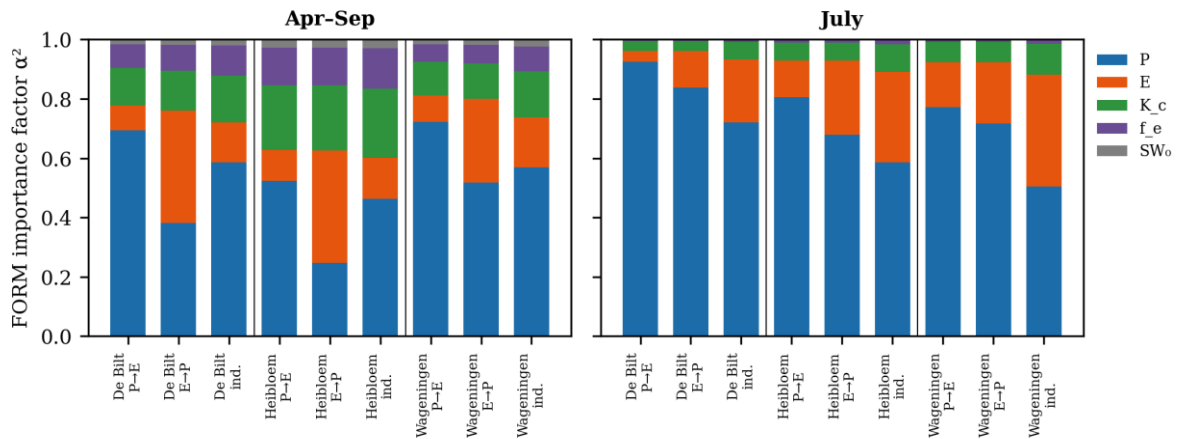


Figure 6. FORM importance factors for the $P \rightarrow E$ and $E \rightarrow P$ Rosenblatt orderings and for independent P and E

3.5. Design Capacities

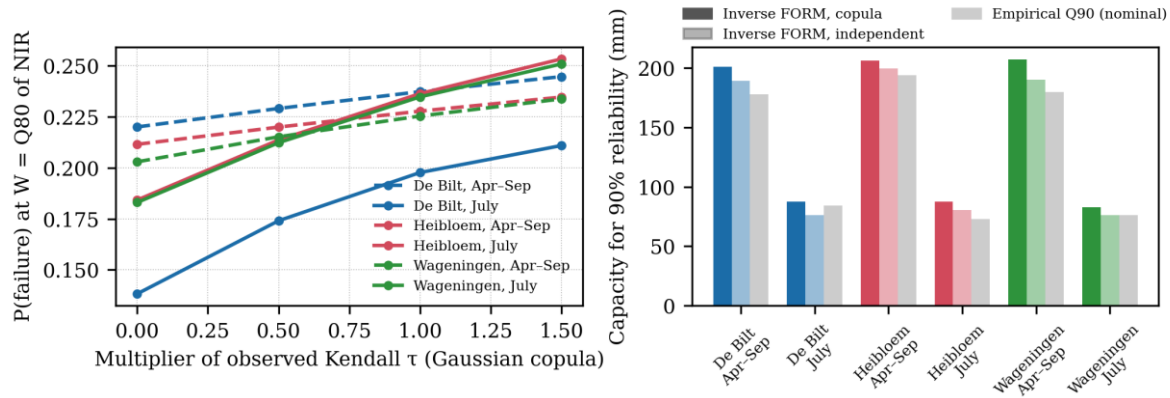
Table 4. Supply capacity (mm) for target reliabilities of 0.80, 0.90 and 0.95: inverse FORM with copula (Cop.) and independence (Ind.), and empirical quantile of the nominal NIR (Emp.)

Site	Period	Cop. 0.80	Ind. 0.80	Emp. 0.80	Cop. 0.90	Ind. 0.90	Emp. 0.90	Cop. 0.95	Ind. 0.95	Emp. 0.95
De Bilt	Apr–Sep	155	148	142	201	189	178	240	223	226
	July	69	62	69	88	76	84	102	87	104
Heibloem	Apr–Sep	171	165	163	206	199	194	234	227	201
	July	74	69	69	88	81	73	98	89	101
Wageningen	Apr–Sep	160	151	156	207	190	180	248	223	196
	July	71	66	67	83	76	76	93	85	91

MCS checks of all inverse-FORM designs gave failure probabilities within 0.019 of the target.

Inverse FORM gave seasonal capacities of 201–207 mm and July capacities of 83–88 mm for 90% reliability (Table 4; Figure 7). MCS checks confirmed that the designs met their targets within 0.019. Designs that ignored dependence were smaller by 3–11% for the season and 7–17% for July, and at most sites the difference increased with the target reliability. At De Bilt, a July supply designed for 95% reliability under independence (87 mm) would be 17% too small. For a 5 ha grassland field this corresponds to about 734 m³ in the peak month.

The reliability-based seasonal capacities for 90% and 95% reliability were larger than the empirical quantiles of the nominal NIR at all sites; for July, they were close to the empirical quantiles. The difference reflects the uncertainty of the crop and soil parameters and the ability of the fitted distributions to extrapolate beyond the short records. The largest difference occurred at Wageningen (95%: 248 against 196 mm), where only 23 seasons were available. Empirical quantiles from short records therefore understate the capacity needed for high reliability.


Figure 7. Failure probability versus strength of P–E dependence (left) and capacities for 90% reliability (right)

3.6. Changes Between Sub-Periods

Table 5. Failure probability at W_0 with the model refitted to the first and second halves of each record

Site	Period	First half	P / E (mm)	τ	P_f	Second half	P / E (mm)	τ	P_f
De Bilt	Apr–Sep	1980–1999	389 / 444	-0.35	0.230	2000–2019	421 / 485	-0.29	0.242
	July	1980–1999	74 / 94	-0.64	0.218	2000–2019	95 / 100	-0.61	0.180
Heibloem	Apr–Sep	1980–1997	359 / 461	-0.39	0.247	1998–2016	405 / 491	-0.32	0.211
	July	1980–1997	64 / 98	-0.54	0.307	1998–2016	80 / 99	-0.18	0.148
Wageningen	Apr–Sep	1976–1986	325 / 420	-0.31	0.225	1987–1999	372 / 454	-0.52	0.215
	July	1976–1987	57 / 86	-0.27	0.193	1988–1999	61 / 96	-0.36	0.245

Between the first and second halves of the records, both P and E increased at every site (Table 5). The two changes largely offset each other: the seasonal failure probability at W_0 changed by less than 0.04 at all sites, and in July it decreased at De Bilt and Heibloem but increased at Wageningen. The half-record samples are short (11–20 years), so these changes are not significant. They show, however, that a design based on one period can be affected by shifts in both variables. Wetter summers alone do not guarantee a lower irrigation risk when evaporative demand increases at the same time (Philip et al., 2020).

3.7. Limitations

The NIR model is a simple seasonal or monthly balance. It ignores the timing of rainfall within the month, capillary rise from shallow groundwater, which is important in parts of the Netherlands, and irrigation efficiency. The parameter distributions for the crop factor, effective rainfall and soil water were assumed rather than measured. The Makkink evaporation at Wageningen was computed from radiation and temperature, whereas the KNMI values were used at the other sites. The records are short for copula selection, and non-stationarity was only examined by splitting the records. Future work could couple the reliability

framework with a daily soil water balance model and use climate projections of P and E.

4. Conclusions

The reliability of supplemental irrigation supply for grassland was assessed at three Dutch sites with a copula-based FORM framework. The main conclusions are:

(1) Precipitation and Makkink evaporation were negatively dependent, weakly for the growing season ($\tau = -0.26$ to -0.17) and significantly for July ($\tau = -0.51$ to -0.35). Seasonal evaporation increased significantly at all sites.

(2) Ignoring the dependence underestimated failure probabilities by 8–43% and the capacity for 95% reliability by up to 17%. The effect was largest for the peak month.

(3) FORM with a Rosenblatt transformation was fast but had errors of up to 10% because of the non-smooth, non-linear limit state. SORM (Hohenbichler–Rackwitz) reduced the error to below 2%, and importance sampling verified the results efficiently.

(4) The Rosenblatt ordering had little effect on the failure probability but changed the importance factors considerably. Precipitation was the most important variable, followed by evaporation and the crop factor.

(5) Empirical percentiles of short NIR records understate the capacity needed for high reliability. Inverse FORM with a fitted copula model provides capacities for explicit reliability targets.

Data Availability Statement

The De Bilt and Heibloem data are distributed with the Pastas package (<https://github.com/pastas/pastas>), originally from KNMI, and the Wageningen weather data with the PCSE package (<https://pypi.org/project/pcse/>). Scripts are available from the corresponding author on request.

Conflict of Interest

The authors declare no conflict of interest.

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