

Copula-Based Joint Return Periods of Drought Duration and Severity at Eleven Climatically Diverse Sites Using a Century of SPEI Data

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Abstract: Drought duration and severity are strongly dependent, so return periods estimated from each variable alone can misrepresent drought risk. This study built bivariate copula models of drought duration (D) and severity (S) at eleven sites on five continents, from arid Lahore to humid Abashiri, using the 6-month Standardized Precipitation Evapotranspiration Index (SPEI-6) computed from 108 years (1900–2007) of monthly climatic water balance. Drought events were identified by run theory (threshold -0.5 , pooling of events separated by one month and removal of one-month events), which gave 50–67 events per site. Kendall's τ between D and S was 0.64 – 0.78 . The lognormal distribution fitted duration best at 10 sites, and severity was described by lognormal, gamma, Weibull or exponential distributions. Gaussian, Clayton, Gumbel and Frank copulas were fitted by maximum likelihood and tested with a Cramér–von Mises test adapted to integer durations. The Clayton copula was rejected at 6 sites. The Gumbel copula, whose upper-tail dependence coefficient (0.69 – 0.83) matched the empirical estimates, was selected at 10 sites. For events whose duration and severity both have 50-year univariate return periods, the joint "AND" return period was 60–76 years, the Kendall return period 55–63 years and the "OR" return period 37–43 years. Assuming independence overestimated the AND return period 16–27-fold (1189–1748 years). Parametric bootstrap showed that the 90% interval of the AND return period spanned a factor of 4–7. The largest observed droughts, including 2001–2004 at Albuquerque and 1997–2000 at Valencia, had Kendall return periods of 50–397 years. Copula choice changed the AND return period of the largest events by up to a factor of 43. More drought events occurred in 1954–2007 than in 1900–1953 at 10 of the 11 sites. Upper-tail-dependent copulas and explicit uncertainty bounds are recommended for drought risk design.

Keywords: Drought, SPEI, Copula, Joint return period, Kendall return period, Tail dependence

1. Introduction

Drought is a recurrent hazard that affects water supply, agriculture and ecosystems on every continent. Unlike floods, droughts develop slowly and are characterised by several correlated properties, mainly duration, severity (cumulative deficit) and intensity (Mishra and Singh, 2010). Run theory, introduced by Yevjevich (1967), identifies a drought event as a run of values below a threshold and provides these properties for frequency analysis. Because long droughts tend to be severe, a univariate return period of either property gives an incomplete description of drought risk (Shiau and Shen, 2001).

Copulas separate the dependence structure of random variables from their marginal distributions (Sklar, 1959; Nelsen, 2006) and have become the standard tool for multivariate hydrological frequency analysis (Genest and Favre, 2007; Salvadori and De Michele, 2004). Shiau (2006) used copulas to model drought duration and severity, and many regional applications followed (Mirabbasi et al., 2012; Madadgar and Moradkhani, 2013; Kao and Govindaraju, 2010). In a bivariate setting, several return periods can be defined. The "OR" return period refers to events in which either variable exceeds its threshold, the "AND" return period to events in which both exceed, and the Kendall return period to

events beyond a critical level of the joint distribution (Salvadori et al., 2011; Gräler et al., 2013). Selecting a design event on a critical layer further requires a rule, such as the most-likely event (Salvadori et al., 2011; Volpi and Fiori, 2012).

Several practical issues remain. Drought duration is an integer number of months, which creates ties that complicate copula goodness-of-fit testing. Copula families differ mainly in their tails, which matter most for extreme droughts, yet copulas are often selected only by likelihood-based criteria. In addition, the sampling uncertainty of joint return periods from records of a few dozen events is rarely reported. Long, homogeneous series from contrasting climates allow these issues to be examined systematically.

The Standardized Precipitation Evapotranspiration Index (SPEI; Vicente-Serrano et al., 2010) includes the effect of evaporative demand and is suitable for drought analysis under warming. The objective of this study was to develop copula-based joint return periods of drought duration and severity at eleven climatically diverse sites using a century of monthly SPEI-6 data. The specific aims were to (i) characterise drought events and their dependence; (ii) select marginal distributions and copulas with tail-sensitive criteria and a tie-aware goodness-of-fit test; (iii) compare OR, AND and

Kendall return periods and the effect of assuming independence; and (iv) quantify the uncertainty due to sampling and copula choice and derive most-likely design droughts.

2. Materials and Methods

2.1. Data and Drought Index

The monthly climatic water balance (precipitation minus potential evapotranspiration) for January 1900 to December 2007 at eleven locations was taken from the example data of the SPEI package (Beguería and Vicente-Serrano, 2023). The data derive from the CRU

TS gridded climate data (Harris et al., 2014), and the series are complete. The sites span arid to humid climates on five continents (Figure 1): Lahore (Pakistan), Kimberley (South Africa), Indore (India), Albuquerque (USA), Punta Arenas (Chile), Valencia (Spain), Tampa (USA), Vienna (Austria), Helsinki (Finland), São Paulo (Brazil) and Abashiri (Japan). In the tables they are ordered from the driest to the wettest mean annual water balance. Coordinates in Figure 1 are approximate city locations.

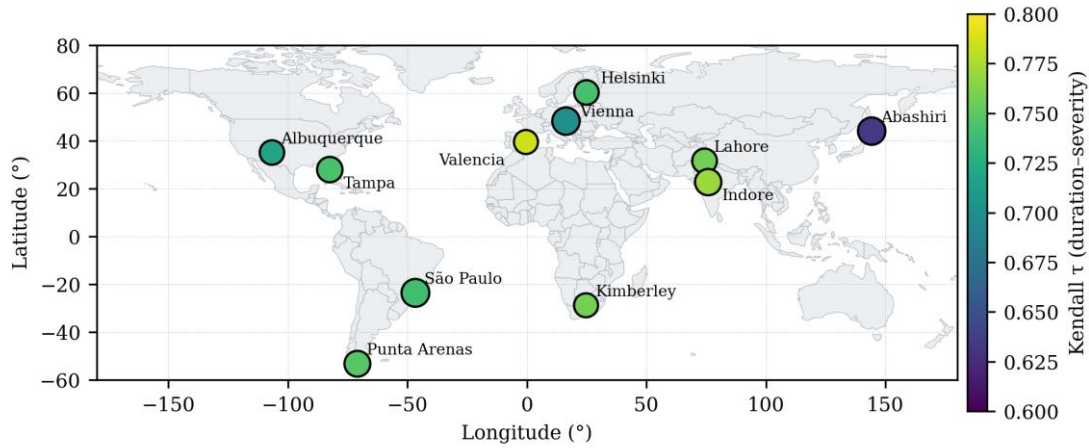


Figure 1. Location of the eleven sites; colour shows Kendall's τ between drought duration and severity, and symbol size the number of events

SPEI-6 was calculated by accumulating the water balance over six months and transforming it to a standard normal variate through a three-parameter log-logistic distribution fitted separately for each calendar month by unbiased probability-weighted moments (Vicente-Serrano et al., 2010; Beguería et al., 2014). The whole record was used as the reference period. The six-month scale represents agricultural and short-term hydrological drought.

2.2. Drought Identification

Drought events were identified by run theory with a threshold of $X_0 = -0.5$ (Figure 2). To avoid splitting one

drought into mutually dependent events, two runs separated by a single month with SPEI-6 still below zero were pooled (Tallaksen et al., 1997). Events lasting only one month after pooling were removed as minor droughts. For each event, the duration D (months from onset to termination) and severity S were recorded:

$$S = \sum_{t=t_s}^{t_e} \max(X_0 - \text{SPEI}_t, 0) \quad (1)$$

where t_s and t_e are the first and last months of the event. The mean inter-arrival time $E(L)$ was estimated as the mean interval between successive onsets.

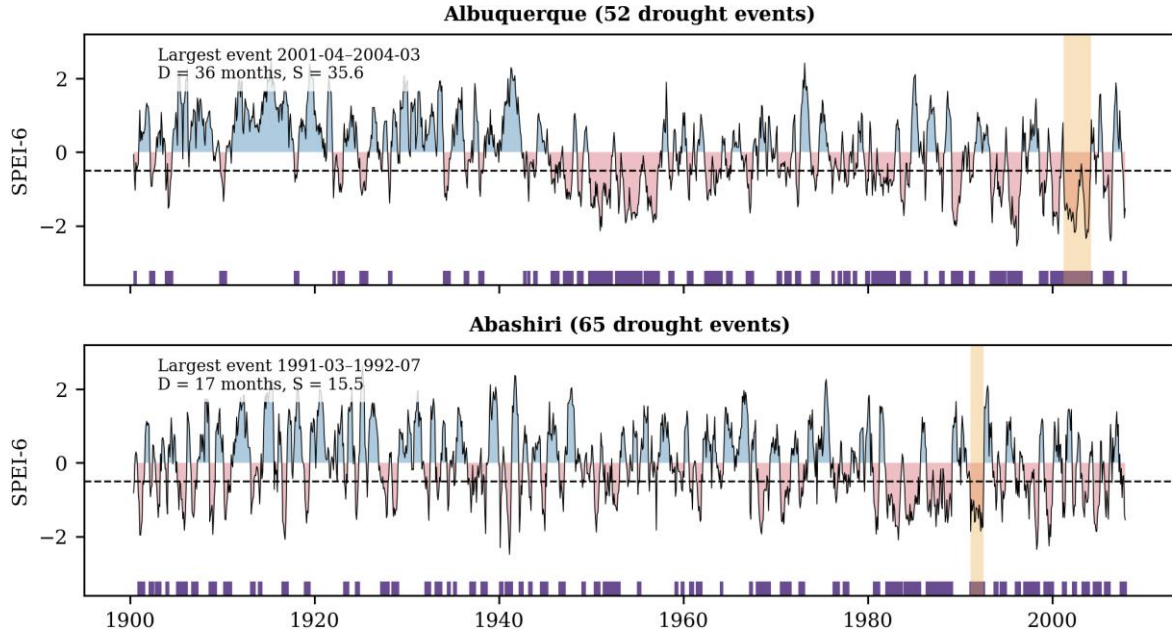


Figure 2. SPEI-6 series and identified drought events (purple bars) at Albuquerque and Abashiri; the largest event is shaded

2.3. Marginal Distributions

Exponential, gamma, Weibull and lognormal distributions (location fixed at zero) were fitted to D and S by maximum likelihood. Because the upper tail governs return periods of extreme droughts, the distribution with the smallest Anderson–Darling (AD) statistic was selected. The AD statistic gives more weight to the tails than the Kolmogorov–Smirnov (KS) test, which was used as an additional check. Duration was treated as continuous for the marginal fit.

2.4. Copula Models

By Sklar's theorem, the joint distribution of D and S can be written as (Sklar, 1959)

$$F_{D,S}(d, s) = C[F_D(d), F_S(s)] \quad (2)$$

where C is a copula. Four one-parameter families were considered (Nelsen, 2006): the Gaussian copula, and the Archimedean Clayton, Gumbel and Frank copulas:

$$C_{Cl}(u, v) = (u^{-\theta} + v^{-\theta} - 1)^{-1/\theta} \quad (3)$$

$$C_{Gu}(u, v) = \exp\left\{-\left[(-\ln u)^\theta + (-\ln v)^\theta\right]^{1/\theta}\right\} \quad (4)$$

$$C_{Fr}(u, v) = -\frac{1}{\theta} \ln \left[1 + \frac{(e^{-\theta u} - 1)(e^{-\theta v} - 1)}{e^{-\theta} - 1} \right] \quad (5)$$

The Gumbel copula has upper-tail dependence $\lambda_U = 2 - 2^{1/\theta}$, the Clayton copula has lower-tail dependence, and the Gaussian and Frank copulas have no tail dependence. The parameters were estimated by maximum pseudo-likelihood from pseudo-observations $u_i = R_i/(n + 1)$, where R_i are ranks with ties averaged (Genest and Favre, 2007). The copula density and conditional distributions were verified numerically against finite differences of the copula functions.

Goodness of fit was tested with the Cramér–von Mises statistic S_n , which compares the empirical copula C_n with the fitted copula (Genest et al., 2009):

$$S_n = \sum_{i=1}^n [C_n(u_i, v_i) - C_{\hat{\theta}}(u_i, v_i)]^2 \quad (6)$$

The p -value was obtained by a parametric bootstrap with 300 replicates. Standard bootstrap samples are continuous and have no ties, whereas observed durations are integers. The bootstrap samples were therefore generated from the fitted copula and marginals, and the simulated durations were rounded to integers (minimum two months) before ranking. This keeps the tie structure of the observed data. Copulas with $p \leq 0.05$ were rejected. Among the remaining copulas, those within four AIC units of the minimum were considered equally supported (Burnham and Anderson, 2002). From these, the copula whose upper-tail dependence was closest to the non-parametric estimate of Frahm et al. (2005) was selected, because joint extremes control drought design.

2.5. Joint Return Periods and Design Events

With $u = F_D(d)$ and $v = F_S(s)$, the univariate, OR, AND and Kendall return periods (in years) are (Shiau, 2006; Salvadori et al., 2011):

$$T_D = \frac{E(L)}{1-u}, \quad T_S = \frac{E(L)}{1-v} \quad (7)$$

$$T_{OR} = \frac{E(L)}{1-C(u,v)} \quad (8)$$

$$T_{AND} = \frac{E(L)}{1-u-v+C(u,v)} \quad (9)$$

$$T_K = \frac{E(L)}{1-K_C[C(u,v)]}, \quad K_C(t) = P[C(U, V) \leq t] \quad (10)$$

where K_C is the Kendall distribution function. For Archimedean copulas, $K_C(t) = t - \varphi(t)/\varphi'(t)$, where φ is the generator. For the Gaussian copula, it was obtained

from 2×10^5 simulated pairs. The return periods were also computed under independence ($C = uv$) to show the effect of ignoring dependence. For design, the most-likely event on the critical layer of a given return period was found by maximising the joint density $f_D(d)f_S(s)c(u,v)$ along the layer (Salvadori et al., 2011; Volpi and Fiori, 2012). The probability that at least one drought exceeding the T -year design event occurs during an N -year design life was computed under a Poisson assumption as $1 - \exp(-N/T)$.

Sampling uncertainty was assessed by a parametric bootstrap (200 replicates per site). Samples of the

observed size were drawn from the fitted model, the marginals and the copula were refitted, and T_{AND} and T_{OR} were recomputed for the event whose duration and severity both equal their 50-year univariate quantiles. The sensitivity to copula choice was evaluated by recomputing the return periods with every non-rejected copula. All computations were coded in Python (NumPy and SciPy).

3. Results and Discussion

3.1. Drought Events

Table 1. Drought event statistics (SPEI-6, threshold -0.5 , 1900–2007)

Site	Events	E(L) (yr)	Mean D (mo)	Max D (mo)	Mean S	Max S	Mean intensity	Kendall τ	Spearman ρ	Events 1900–53 / 1954–2007
Lahore	53	2.06	7.4	27	4.80	22.4	0.55	0.76	0.90	25 / 28
Kimberley	50	1.90	8.3	26	5.19	20.1	0.54	0.76	0.90	21 / 29
Indore	60	1.81	6.9	23	4.33	22.5	0.50	0.77	0.92	24 / 36
Albuquerque	52	2.10	8.3	36	4.94	35.6	0.47	0.72	0.87	21 / 31
Punta Arenas	58	1.87	7.5	31	4.38	27.9	0.50	0.75	0.89	18 / 40
Valencia	50	2.09	8.1	40	5.09	34.9	0.51	0.78	0.91	23 / 27
Tampa	56	1.82	7.6	31	4.56	31.8	0.51	0.74	0.90	24 / 32
Vienna	65	1.65	6.5	27	3.89	21.9	0.53	0.70	0.87	28 / 37
Helsinki	52	2.08	8.0	29	5.09	23.9	0.57	0.74	0.89	23 / 29
São Paulo	67	1.43	6.1	19	3.74	22.8	0.50	0.74	0.89	28 / 39
Abashiri	65	1.67	6.6	21	3.91	15.5	0.56	0.64	0.80	33 / 32

Between 50 (Kimberley and Valencia) and 67 (São Paulo) drought events were identified, corresponding to mean inter-arrival times of 1.43–2.10 years (Table 1). Mean durations were 6.1–8.3 months, and the longest droughts lasted 40 months at Valencia (June 1997 to September 2000) and 36 months at Albuquerque (April 2001 to March 2004). Duration and severity were strongly and positively dependent (Figure 3). Kendall's τ was 0.64 at Abashiri and 0.70–0.78 elsewhere. Such strong dependence is typical of accumulated deficits, because a longer event accumulates more deficit.

At 10 of the 11 sites, more events started in 1954–2007 than in 1900–1953. Over all sites, the number rose from 268 to 360 (binomial test $p = 0.0003$, treating events as independent). The largest increase occurred at Punta Arenas (18 to 40 events). This is consistent with the drying trends in SPEI reported for many regions and attributed partly to increasing evaporative demand (Vicente-Serrano et al., 2014). Because the SPEI was standardised on the whole record, the copula models describe the average behaviour of the century. Return periods for present conditions may therefore be shorter.

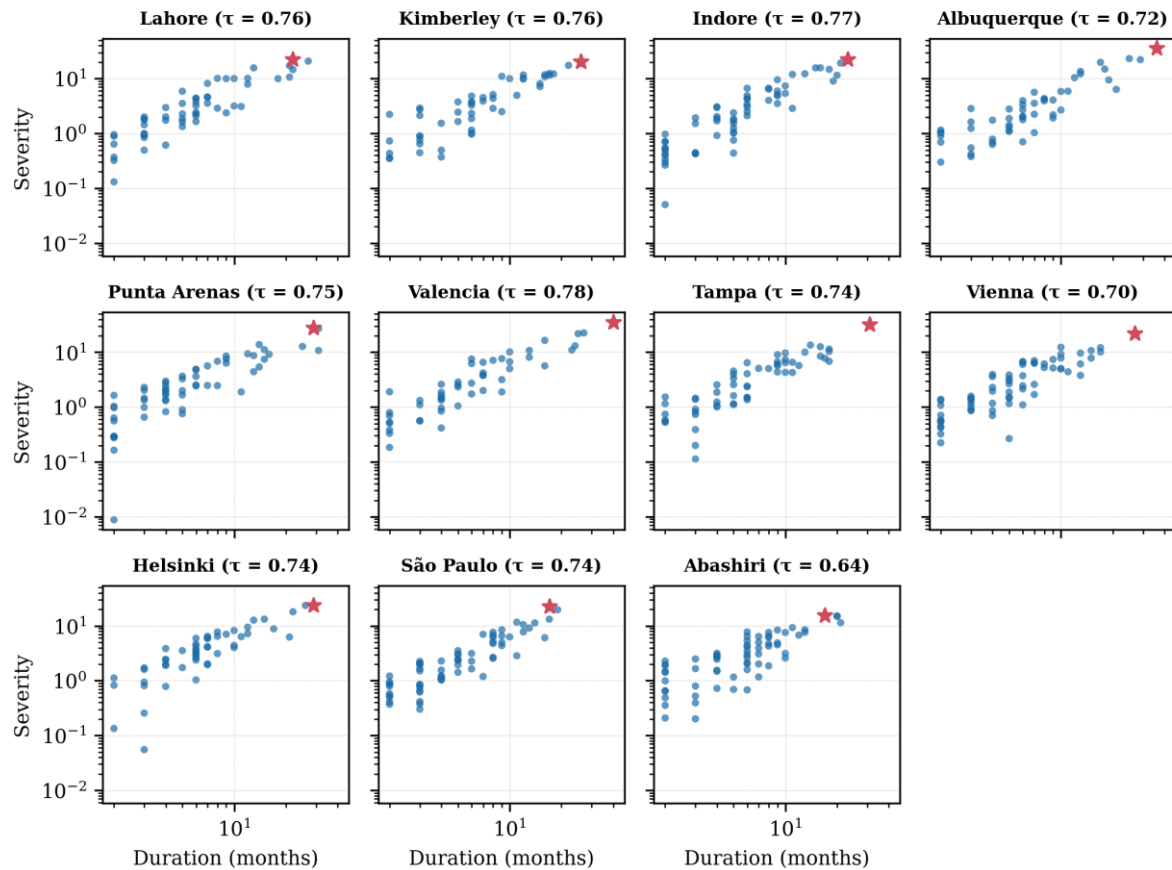


Figure 3. Drought duration versus severity at the eleven sites (log scales); the star marks the event with the largest severity

3.2. Marginal Distributions

Table 2. Selected marginal distributions (minimum Anderson–Darling statistic)

Site	Duration	AD	KS p	Severity	AD	KS p
Lahore	Lognormal	0.69	0.50	Lognormal	0.30	0.82
Kimberley	Lognormal	0.61	0.47	Lognormal	0.64	0.54
Indore	Lognormal	0.94	0.33	Lognormal	0.39	0.88
Albuquerque	Lognormal	0.45	0.83	Lognormal	0.41	0.90
Punta Arenas	Lognormal	1.05	0.33	Weibull	0.49	0.51
Valencia	Lognormal	0.69	0.52	Lognormal	0.20	0.96
Tampa	Lognormal	0.58	0.61	Weibull	0.52	0.46
Vienna	Lognormal	0.75	0.51	Exponential	0.54	0.74
Helsinki	Lognormal	0.65	0.36	Gamma	0.40	0.94
São Paulo	Lognormal	1.16	0.11	Lognormal	0.41	0.80
Abashiri	Gamma	0.97	0.41	Gamma	0.25	0.87

The lognormal distribution was selected for duration at 10 sites and the gamma distribution at Abashiri (Table 2; Figure 4). The exponential distribution, often assumed for drought duration, had the largest AD statistic at every site. For severity, the lognormal was selected at 6 sites, and the gamma, Weibull and exponential at the others. All selected distributions passed the KS test ($p \geq 0.11$). The candidate severity distributions differed mainly in the upper tail, which is where univariate return periods beyond the record length are extrapolated.

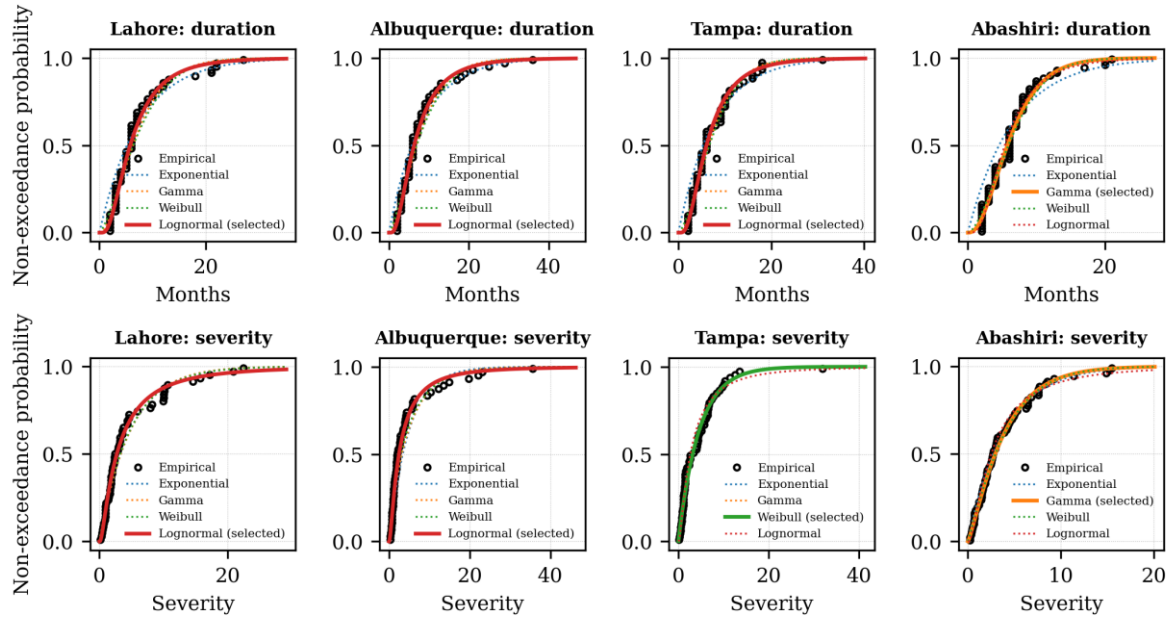


Figure 4. Empirical and fitted marginal distributions of duration (top) and severity (bottom) at four representative sites

3.3. Copula Selection

Table 3. Copula selection: ΔAIC of each family, selected copula, parameter, model τ , CvM p-value and upper-tail dependence

Site	ΔAIC Gauss.	ΔAIC Clayton	ΔAIC Gumbel	ΔAIC Frank	Selected	θ	τ (model)	CvM p	λ_U empirical	λ_U model
Lahore	0.0	25.1	4.5	9.4	Gaussian	0.918	0.74	0.430	0.77	0.00
Kimberley	10.3	40.7	0.0	15.5	Gumbel	3.981	0.75	0.184	0.81	0.81
Indore	2.5	39.9	0.0	7.4	Gumbel	3.821	0.74	0.360	0.80	0.80
Albuquerque	9.3	41.5	0.0	11.8	Gumbel	3.294	0.70	0.500	0.77	0.77
Punta Arenas	0.0	26.6	3.1	6.0	Gumbel	3.353	0.70	0.297	0.77	0.77
Valencia	9.0	37.6	0.0	12.8	Gumbel	4.305	0.77	0.390	0.83	0.83
Tampa	2.9	32.3	2.4	0.0	Gumbel	3.192	0.69	0.407	0.76	0.76
Vienna	1.5	24.0	0.0	3.8	Gumbel	3.025	0.67	0.404	0.74	0.74
Helsinki	4.1	31.5	0.0	11.1	Gumbel	3.568	0.72	0.091	0.79	0.79
São Paulo	6.2	44.3	0.0	6.6	Gumbel	3.294	0.70	0.337	0.76	0.77
Abashiri	2.1	27.6	0.0	7.7	Gumbel	2.571	0.61	0.145	0.69	0.69

ΔAIC = AIC – minimum AIC at the site; CvM = Cramér–von Mises test with tie-preserving parametric bootstrap (300 replicates); λ_U empirical = non-parametric estimator of Frahm et al. (2005).

The Clayton copula fitted worst at every site (ΔAIC = 24–44) and was rejected by the CvM test at 6 sites (Table 3; Figure 5). Its lower-tail dependence does not describe drought data, in which the strongest association occurs between long and severe events. No other copula was rejected ($p \geq 0.071$). The Gumbel copula had the lowest AIC at 8 sites, the Gaussian copula at Lahore and Punta Arenas and the Frank copula at Tampa. At Punta Arenas and Tampa, however, the Gumbel copula was within four AIC units of the best model. Its upper-tail dependence matched the empirical estimate (0.77

against 0.77 at Punta Arenas; 0.76 against 0.76 at Tampa), whereas the Gaussian and Frank copulas imply none. The Gumbel copula was therefore selected at ten sites, and the Gaussian copula at Lahore, where it was clearly better (ΔAIC = 4.5 for Gumbel).

The empirical upper-tail dependence coefficients were 0.69–0.83, and the Gumbel values were within 0.01 of them. The fitted Gumbel parameters (θ = 2.57–4.30) reproduced Kendall's τ closely, and the fitted and empirical copula level curves agreed well (Figure 6).

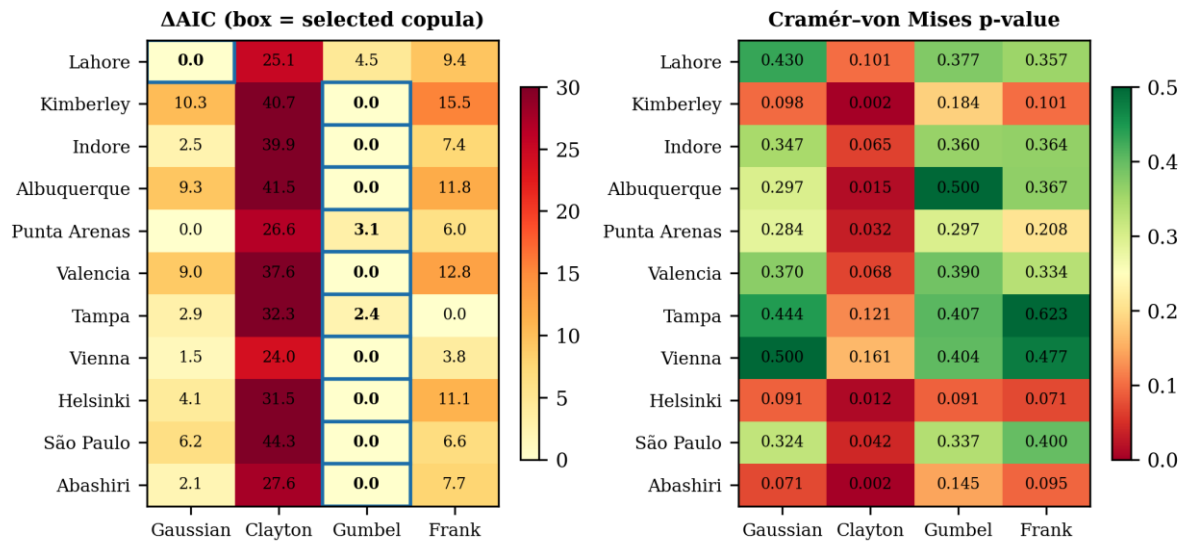
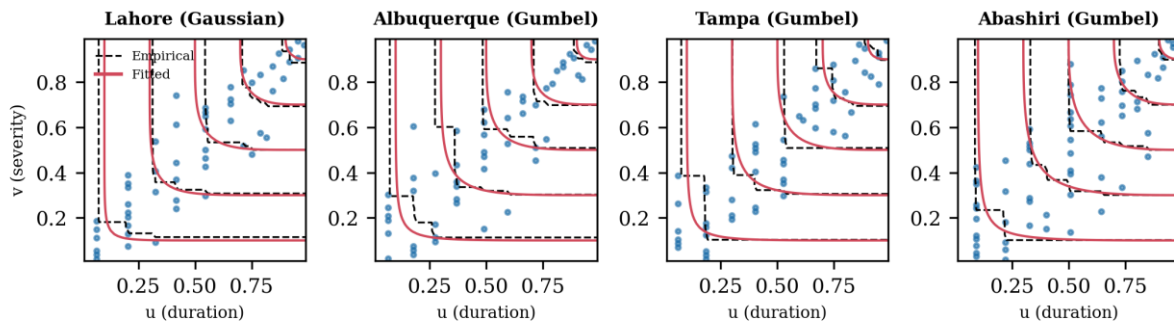
Figure 5. Copula selection: Δ AIC (selected copula boxed) and Cramér-von Mises p-values

Figure 6. Level curves of the empirical (dashed) and selected fitted (solid) copulas with pseudo-observations

3.4. Joint Return Periods

Table 4. Joint return periods (years) of events whose duration and severity equal their 50- and 100-year univariate quantiles

Site	d / s (T = 50)	T _{OR}	T _K	T _{AND}	T _{AND} 90% CI	T _{AND} indep.	d / s (T = 100)	T _{OR}	T _K	T _{AND}
Lahore	19.2 / 19.1	37.3	62.7	75.9	41–192	1213	23.7 / 26.9	72.4	131.8	161.4
Kimberley	23.1 / 23.2	42.2	55.9	61.4	33–218	1315	28.7 / 32.9	84.2	112.0	123.2
Indore	18.8 / 21.9	41.9	56.3	62.1	31–182	1382	23.2 / 32.3	83.6	112.7	124.5
Albuquerque	21.9 / 17.6	40.7	57.8	64.8	34–218	1189	27.3 / 24.9	81.2	116.0	130.1
Punta Arenas	21.4 / 15.8	40.8	57.6	64.5	33–207	1340	26.9 / 19.6	81.5	115.6	129.4
Valencia	22.1 / 20.8	42.7	55.2	60.3	32–190	1199	28.0 / 30.2	85.3	110.7	120.9
Tampa	20.6 / 15.2	40.4	58.3	65.5	36–201	1374	25.2 / 18.3	80.7	116.8	131.5
Vienna	17.3 / 13.3	39.9	59.0	66.9	38–162	1518	20.9 / 16.0	79.7	118.4	134.2
Helsinki	19.1 / 15.3	41.4	56.9	63.2	35–210	1202	23.2 / 18.5	82.5	114.1	126.9
São Paulo	16.7 / 15.9	40.6	57.9	64.9	31–168	1748	20.0 / 21.5	81.2	116.1	130.2
Abashiri	15.6 / 12.1	38.4	61.9	71.7	34–206	1496	17.8 / 14.4	76.6	124.4	144.1

d in months; CI from 200 parametric bootstrap replicates; indep. = independence copula.

For an event with 50-year duration and 50-year severity, the joint return periods followed the expected order $T_{OR} < T_K < T_{AND}$ (Table 4; Figure 8). T_{OR} was 37–43 years, T_K 55–63 years and T_{AND} 60–76 years. Because of the strong dependence, all three were close to the univariate value of 50 years. The OR and AND values would coincide with it under perfect dependence. Assuming independence gave very different results: T_{AND} rose to 1189–1748 years, 16–27 times the copula value, and T_{OR} fell to 25.4–25.5 years. A design based on independence would therefore greatly underestimate the frequency of droughts that are both long and severe.

The bootstrap intervals were wide. The 90% interval of T_{AND} spanned 31–41 years (lower bounds) to 162–218 years (upper bounds), a width factor of 4.3–6.7, even though each site had more than 50 events from a 108-year record. T_{OR} was less uncertain (20–122 years over all sites). The Kendall return periods lay between the OR and AND values at all return levels, consistent with Salvadori et al. (2011). Joint return period contours in the duration–severity plane show how the AND and OR definitions bound the region of equally rare events (Figure 7).

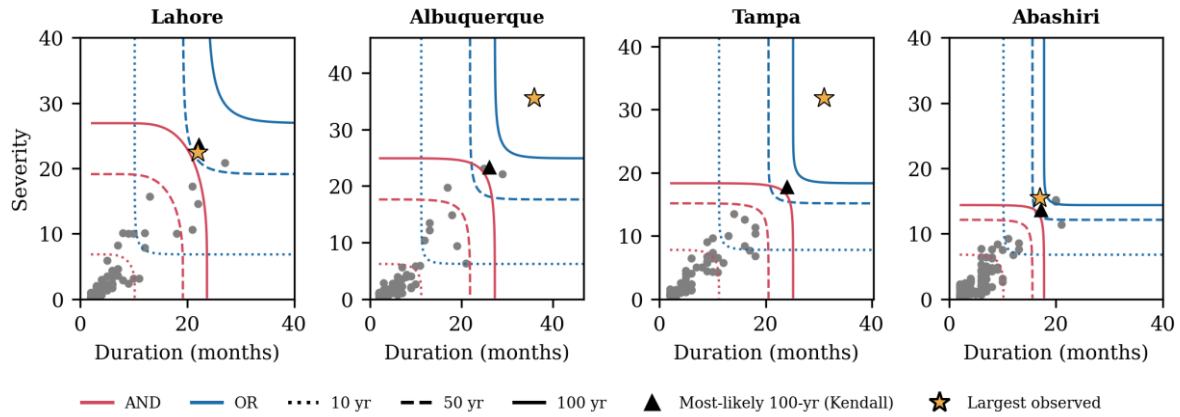


Figure 7. AND and OR joint return period contours (10, 50 and 100 years), observed events, the most-likely 100-year Kendall design event and the largest observed event

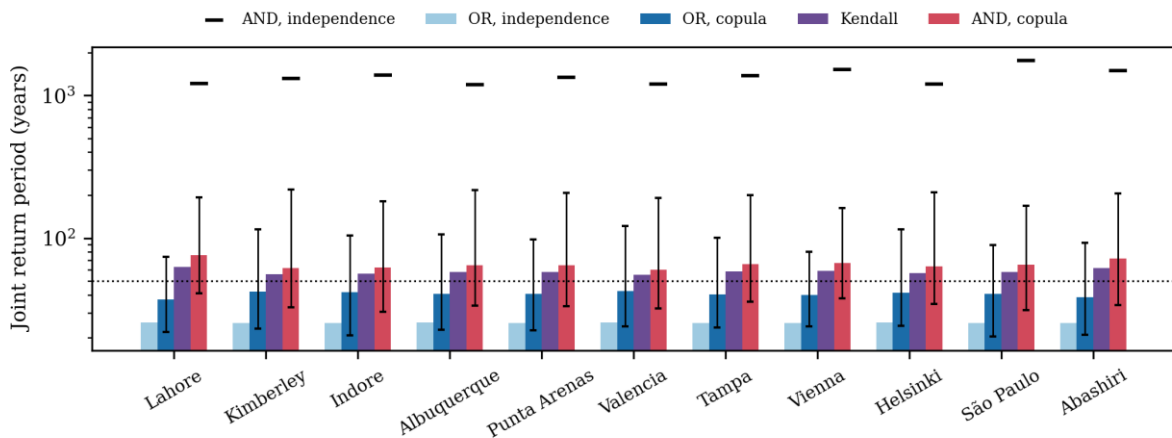


Figure 8. Joint return periods of the event with 50-year univariate duration and severity: OR, Kendall and AND (with 90% bootstrap intervals) and the independence assumption

3.5. Largest Observed Droughts

Table 5. Largest observed drought (maximum severity) at each site and its return periods (years)

Site	Period	D (mo)	S	Min SPEI-6	T _D	T _S	T _{OR}	T _K	T _{AND}
Lahore	1946-03 – 1947-12	22	22.4	-3.10	78	68	53	93	115
Kimberley	1982-12 – 1985-01	26	20.1	-2.22	72	39	38	50	75
Indore	2001-08 – 2003-06	23	22.5	-2.45	97	52	51	69	101
Albuquerque	2001-04 – 2004-03	36	35.6	-2.35	266	222	195	279	320
Punta Arenas	1936-06 – 1938-10	29	27.9	-2.16	127	425	127	180	432
Valencia	1997-06 – 2000-09	40	34.9	-2.31	338	134	134	174	342
Tampa	1998-08 – 2001-02	31	31.8	-2.44	221	1890	220	320	1895
Vienna	2000-05 – 2002-07	27	21.9	-2.26	286	455	266	397	516
Helsinki	1938-06 – 1940-10	29	23.9	-2.15	251	330	230	319	376
São Paulo	1963-04 – 1964-08	17	22.8	-2.67	54	115	52	75	121
Abashiri	1991-03 – 1992-07	17	15.5	-1.86	78	139	72	117	163

The most severe droughts occurred in different decades at different sites (Table 5): the 1930s at Punta Arenas and Helsinki, 1946–1947 at Lahore, 1963–1964 at São Paulo, 1982–1985 at Kimberley, 1991–1992 at Abashiri and 1997–2004 at Valencia, Tampa, Vienna, Indore and Albuquerque. Their Kendall return periods ranged from 50 years (Kimberley) to 397 years (Vienna). Univariate return periods of the same events differed strongly between duration and severity. At Tampa, the 1998–2001 drought had a duration return period of 221 years but a severity return period of 1890 years. This extrapolation far beyond the 108-year record reflects the light upper tail of the fitted Weibull severity

distribution. Such values should be treated as indicative only, since the marginal tails are poorly constrained by about 50 events.

The multi-year droughts of 1997–2004 in the Mediterranean, central Europe and the south-western USA stand out: the Kendall return periods were 174 years at Valencia, 397 years at Vienna and 279 years at Albuquerque. This agrees with reports of increasing drought severity in southern Europe under rising temperature (Vicente-Serrano et al., 2014).

3.6. Sensitivity to Copula Choice

Copula choice had a large effect on joint return periods, especially for the AND definition (Figure 9).

For the 50-year univariate event, T_{AND} was 60–72 years with the Gumbel copula, 74–100 years with the Gaussian, 133–237 years with the Frank and 323–459 years with the Clayton copula (where not rejected). For the largest observed events, the ratio between the largest and smallest estimates reached 43 (Vienna). At Tampa, for example, T_{AND} was 1895 years with the Gumbel and

20341 years with the Frank copula, although the two fitted the data almost equally well ($\Delta AIC = 2.4$). The OR return periods were much less sensitive. Copulas without upper-tail dependence make joint extremes too rare, so model selection by likelihood alone can lead to unsafe designs. Checking tail dependence, as done here, is a simple safeguard.

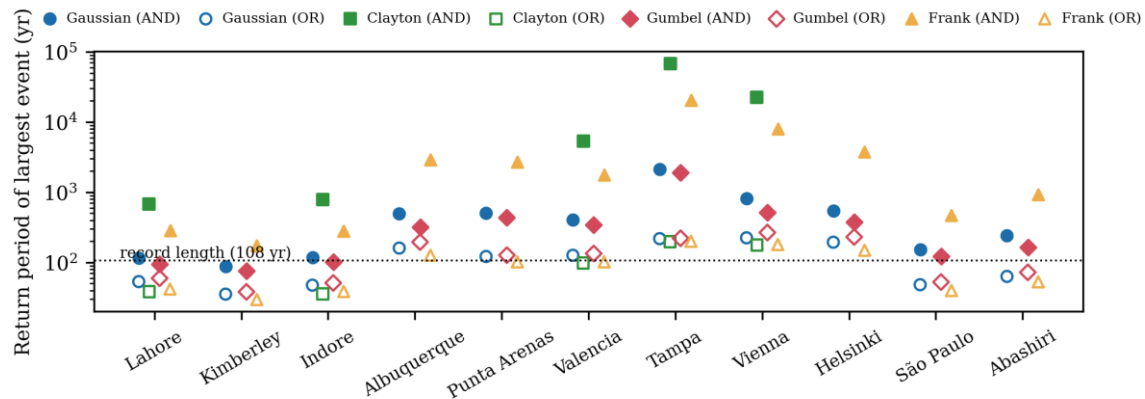


Figure 9. AND (filled) and OR (open) return periods of the largest observed drought computed with each non-rejected copula

3.7. Design Droughts and Risk

Table 6. Most-likely 100-year design droughts (duration in months / severity) for the OR, Kendall and AND definitions

Site	OR	Kendall	AND
Lahore	26.1 / 31.1	22.2 / 23.5	21.2 / 21.0
Kimberley	30.2 / 35.7	27.6 / 31.4	27.1 / 29.4
Indore	24.7 / 34.8	22.5 / 30.1	21.7 / 29.0
Albuquerque	29.0 / 27.7	26.2 / 23.2	25.3 / 21.9
Punta Arenas	28.3 / 21.1	25.7 / 18.8	24.7 / 18.3
Valencia	29.7 / 32.3	27.2 / 28.6	26.1 / 27.7
Tampa	26.8 / 19.3	24.0 / 17.7	23.2 / 17.2
Vienna	22.2 / 16.9	19.8 / 15.5	19.1 / 15.0
Helsinki	24.3 / 19.4	22.2 / 18.0	21.6 / 17.5
São Paulo	21.0 / 23.5	19.5 / 19.9	18.7 / 19.3
Abashiri	18.5 / 15.3	17.2 / 13.6	16.6 / 13.2

The most-likely 100-year Kendall design drought lasted 17–28 months, with severities of 13.6–31.4 (Table 6). The OR definition gave the most demanding design events and the AND definition the least demanding, with OR durations 11–23% longer than AND durations. The design events lay close to the diagonal of the copula space ($u \approx v$) because of the strong dependence. The choice of definition should follow the failure mechanism of the system: OR if either a long or a severe drought causes failure, and AND if failure requires both.

Return periods must also be interpreted as risks over a design life. Under the Poisson assumption, the probability that a 100-year design drought is exceeded at least once in 50 years is 0.39, and for a 50-year design it is 0.63. Combined with the bootstrap uncertainty of the AND return period, a system designed for a nominal 100-year joint drought may in fact be designed for a considerably more frequent event. Designers should therefore use upper confidence bounds or a safety margin.

3.8. Limitations

The analysis used a single drought index at one time scale and a single threshold, and the results depend on the pooling and minor-event criteria. The data are gridded products, and early-century values rely on sparse station networks. Stationarity was assumed, although event frequency increased in the second half of the century. Only one-parameter bivariate copulas were considered; intensity or peak could be added in a trivariate or vine copula framework. Future work could use non-stationary copulas with time-varying marginals and combine several indices and time scales.

4. Conclusions

Bivariate copula models of drought duration and severity were developed for eleven sites using 108 years of SPEI-6. The main conclusions are:

(1) Duration and severity were strongly dependent ($\tau = 0.64$ – 0.78). Durations were best described by the lognormal distribution. The number of events increased in 1954–2007 compared with 1900–1953.

(2) The Clayton copula was inadequate. The Gumbel copula, with upper-tail dependence matching the data ($\lambda_U = 0.69$ – 0.83), was selected at ten sites. A tie-aware bootstrap is needed to test copula fit when durations are integers.

(3) For events with 50-year univariate duration and severity, $T_{OR} = 37$ – 43 , $T_K = 55$ – 63 and $T_{AND} = 60$ – 76 years. Ignoring dependence overestimated T_{AND} 16–27-fold.

(4) Sampling uncertainty (a factor of 4–7 for T_{AND}) and copula choice (up to a factor of 43 for the largest events) dominate the uncertainty of joint return periods. Both should be reported in drought risk assessments.

(5) Most-likely design droughts differ between the OR, Kendall and AND definitions, and the definition

should match the failure mechanism of the water system being designed.

Data Availability Statement

The climatic water balance data are distributed with the SPEI R package (<https://CRAN.R-project.org/package=SPEI>). The SPEI-6 series, event lists and scripts are available from the corresponding author on request.

Conflict of Interest

The authors declare no conflict of interest.

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