

Frequency Analysis of Monthly Rainfall of a Hyper-Arid Region over Iran Using the Exponential Distribution

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Abstract: Frequency analysis of rainfall is a fundamental tool for the planning, design and operation of water resource, irrigation and drainage systems, particularly in arid and hyper-arid regions where rainfall is scarce, highly variable and frequently equal to zero in several months of the year. This study evaluates the one-parameter exponential distribution as a model for the frequency analysis of monthly rainfall in Zabol, a hyper-arid station in south-eastern Iran, using 52 years (1969-2020) of monthly rainfall records from the Zabol synoptic station. The single parameter of the exponential distribution, α , was estimated by the method of moments as the reciprocal of the mean monthly rainfall, and the probability density, cumulative distribution and quantile functions were derived for each month. Goodness of fit was assessed using the mean relative deviation (MRD) and the mean square relative deviation (MSRD) between observed and computed values, and the resulting quantiles were compared with those obtained from the two-parameter gamma and log-Pearson type III distributions, which are more commonly used in hydrological frequency analysis. The exponential distribution was found to describe the monthly rainfall of Zabol reasonably well, particularly once months with a high proportion of near-zero observations were treated separately, and its quantile estimates were broadly comparable with those of the two-parameter gamma and log-Pearson type III distributions while requiring the estimation of only a single parameter. Given its mathematical simplicity, the exponential distribution is proposed as a practical, low-data alternative for the frequency analysis of monthly rainfall in hyper-arid environments, complementing rather than replacing more heavily parameterised distributions.

Keywords: Probability distribution function, Log-Pearson type III distribution, Two-parameter gamma distribution, Parameter estimation, One-parameter distribution, Hyper-arid rainfall

1. Introduction

Rainfall is the primary input to almost every hydrological and water-resource planning problem, and a reliable description of its frequency distribution is essential for the design of irrigation and drainage systems, the sizing of water-harvesting and storage structures, and the assessment of agricultural and hydrological drought risk (Linsley et al., 1982; Sharma, 1996). In arid and hyper-arid regions, this task is complicated by the fact that monthly rainfall is frequently very low, highly skewed and, in several months of the year, equal to zero at a large proportion of stations, so that distributions developed for humid climates often perform poorly or require awkward zero-inflation corrections (Sen and Eljadid, 1999; Modarres, 2006).

A wide range of probability distributions has been proposed for rainfall frequency analysis, including the normal, log-normal, gamma, Pearson type III, log-Pearson type III, Gumbel and Weibull distributions, most of which require the estimation of two or three parameters (Gregory, 1969; Mooley and Appa Rao, 1971; Eagleson, 1972). Two- and three-parameter

distributions can represent a wider range of shapes, but they also demand longer, more reliable records to estimate their parameters stably, a requirement that is often difficult to satisfy at rainfall stations in remote or data-scarce hyper-arid regions such as south-eastern Iran (Khalili, 1973, 1976; Haghighat jou, 1989, 2002). Iran has an average annual rainfall of roughly one-third of the global average, and its eastern and south-eastern provinces, including Sistan and Baluchestan where Zabol is located, are among the most rainfall-scarce parts of the country (Abolverdi and Khalili, 2010; Haghighat jou et al., 2009).

The one-parameter exponential distribution offers a mathematically simple alternative. As a special case of both the gamma and the Weibull distributions, it requires the estimation of only a single parameter, which under the method of moments is simply the reciprocal of the mean, and its probability density, cumulative distribution and quantile functions all have closed analytical forms (Brase and Brase, 2012). Despite this simplicity, and despite its long-standing use in reliability engineering and in the analysis of inter-arrival times of hydrological events such as storms and floods, the

exponential distribution has been little used for the direct frequency analysis of monthly rainfall totals, in Iran or, to the authors' knowledge, elsewhere (Alamatsaz and Bidram, 2007; Haghighat jou et al., 2013).

The present study therefore evaluates the exponential distribution as a candidate model for the frequency analysis of monthly rainfall at Zabol, a representative hyper-arid station in south-eastern Iran, using a 52-year monthly rainfall record. The specific objectives are: (i) to fit the one-parameter exponential distribution to the monthly rainfall series and derive its probability density, cumulative distribution and quantile functions; (ii) to assess the goodness of fit of the exponential distribution using the mean relative deviation and mean square relative deviation between observed and computed values; and (iii) to compare the resulting rainfall quantiles with those obtained from the two-parameter gamma and log-Pearson type III distributions, which are more commonly applied in hydrological practice.

2. Materials and Methods

2.1. Study Area and Data

The study uses monthly rainfall data recorded at the Zabol synoptic station, Sistan and Baluchestan province, south-eastern Iran, over the 52-year period 1969-2020, obtained from the meteorological almanacs of the Islamic Republic of Iran Meteorological Organization. Zabol has a hyper-arid climate characterised by low and highly variable rainfall, and the months of June to September were excluded from the analysis because rainfall in this period is zero, or negligibly different from zero, in almost every year of record. The descriptive statistics of monthly rainfall for the remaining eight months are summarised in Table 1.

Table 1. Statistical characteristics of monthly precipitation (mm) at the Zabol synoptic station, 1969-2020

Month	Min.	Median	Max.	Mean	SD	C _s
January	0.0	11.0	70.0	14.5	13.5	1.89
February	0.0	6.8	59.0	13.1	14.4	1.52
March	0.0	7.4	74.9	15.6	18.2	1.48
April	0.0	2.0	33.4	5.9	8.1	1.70
May	0.0	0.0	11.0	0.6	1.9	4.30
October	0.0	0.0	63.4	2.2	9.9	5.89
November	0.0	0.0	20.0	2.7	3.4	1.98
December	0.0	1.8	48.6	7.4	11.6	2.10

C_s: coefficient of skewness.

2.2. The Exponential Distribution

The exponential distribution is a one-parameter continuous probability distribution, and a special case of both the two-parameter gamma and Weibull distributions, whose probability density function is

$$f(x) = \alpha e^{-\alpha x}, \quad 0 < x < \infty, \quad \alpha > 0 \quad (1)$$

where x is the monthly rainfall depth and α is the single parameter of the distribution, estimated in this study by the method of moments as the reciprocal of the sample

mean, $\alpha = 1 / \bar{x}$. The corresponding cumulative distribution function is

$$F(x) = 1 - e^{-\alpha x} \quad (2)$$

and the quantile (inverse cumulative distribution) function, used to estimate the rainfall depth associated with a given non-exceedance probability $F(x)$, is obtained by inverting Eq. (2):

$$x = -(1 / \alpha) \cdot \ln[1 - F(x)] \quad (3)$$

The main theoretical characteristics of the exponential distribution, expressed in terms of the parameter α , are summarised in Table 2; notably, the coefficient of variation of an exponential variate is always equal to 1, regardless of the value of α , which provides a simple diagnostic check on the appropriateness of the distribution for a given data set.

Table 2. Theoretical characteristics of the exponential distribution with parameter α

Statistic	Expression
Mean	$1 / \alpha$
Variance	$1 / \alpha^2$
Standard deviation	$1 / \alpha$
Coefficient of variation	1
Coefficient of skewness	2
Coefficient of kurtosis	9

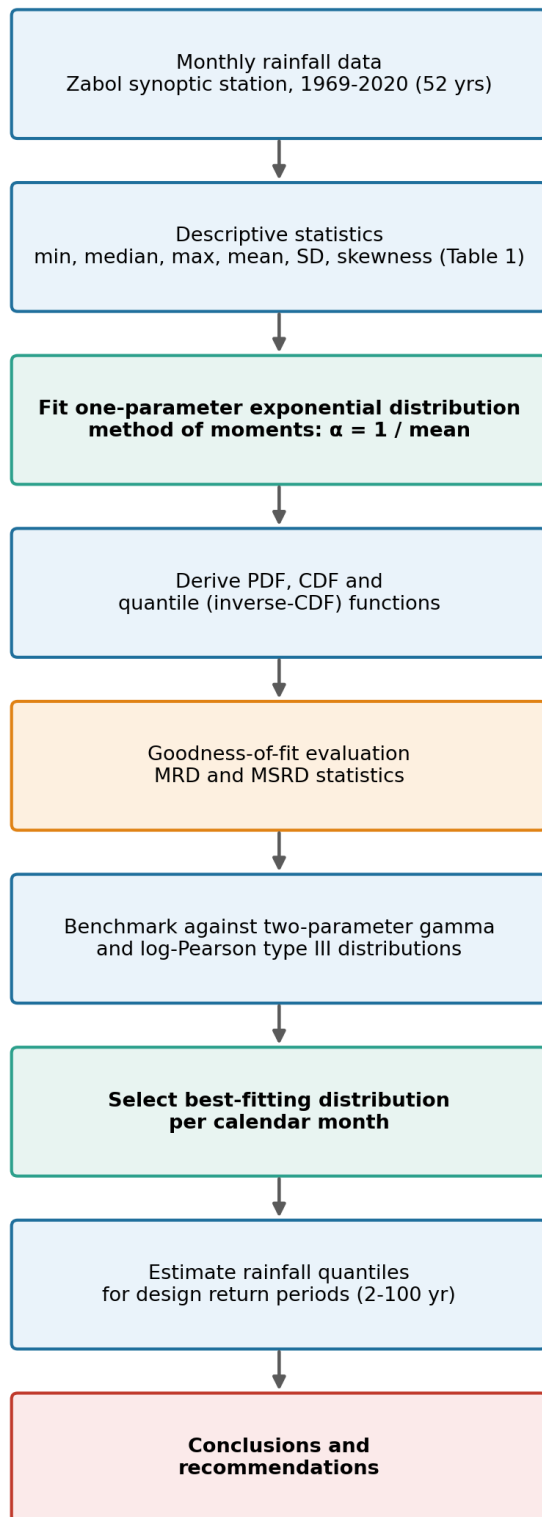


Figure 1. Methodological flowchart of the frequency analysis of monthly rainfall using the exponential distribution

2.3. Goodness-of-Fit Assessment

The goodness of fit of the exponential distribution, and of the benchmark gamma and log-Pearson type III distributions, was evaluated using the mean relative deviation (MRD) and the mean square relative deviation

(MSRD) between the observed and the distribution-computed values:

$$MRD = (1 / n) \sum |X_{oi} - X_{ci}| / X_{oi} \quad (4)$$

$$MSRD = (1 / n) \sum [(X_{oi} - X_{ci}) / X_{oi}]^2 \quad (5)$$

where X_{oi} and X_{ci} are, respectively, the observed and the distribution-computed values of the i -th ranked observation, and n is the sample size. Lower values of MRD and MSRD indicate a closer correspondence between the observed rainfall series and the fitted distribution. The exponential-distribution quantiles were further compared, at return periods of 2, 5, 10, 20, 25, 50 and 100 years, with quantiles obtained from the two-parameter gamma and log-Pearson type III distributions fitted to the same monthly series, which are more widely used in hydrological practice and served here as a benchmark of practical adequacy. Parameters of the gamma and log-Pearson type III distributions were estimated by conventional moment-based procedures, and general frequency-analysis theory and regression diagnostics followed standard references (Kite, 1978; Neter et al., 1988; Rao and Hamed, 2000; Singh, 1998).

3. Results and Discussion

Figures 2 and 3 show the probability density and cumulative distribution functions of the exponential distribution for three illustrative values of the parameter α (0.5, 1.0 and 1.5), which reproduce the characteristic monotonically decreasing density and the concave cumulative curve of the distribution. As α increases, the distribution becomes more concentrated near the origin, consistent with drier months in which the mean monthly rainfall, and hence $1 / \alpha$, is small.

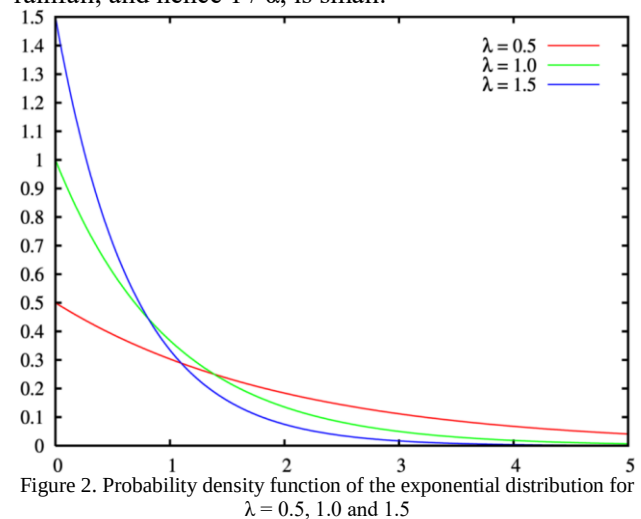


Figure 2. Probability density function of the exponential distribution for $\lambda = 0.5, 1.0$ and 1.5

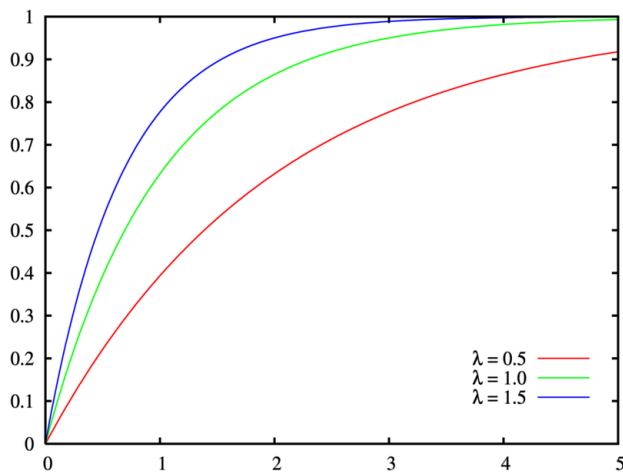


Figure 3. Cumulative distribution function of the exponential distribution for $\lambda = 0.5, 1.0$ and 1.5

The observed frequency distributions of monthly rainfall for January to April, the four wettest months at Zabol, are shown in Figures 4-7. In every case the distribution is strongly right-skewed, with a large proportion of months receiving less than 10 mm of rainfall and a long tail of infrequent wetter months, a shape consistent with the exponential probability density function.

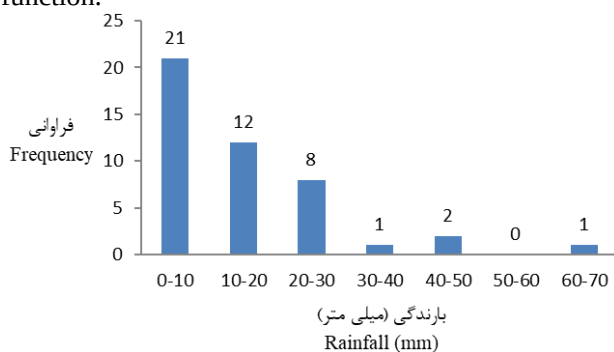


Figure 4. Observed frequency distribution of January rainfall at Zabol, 1969-2020

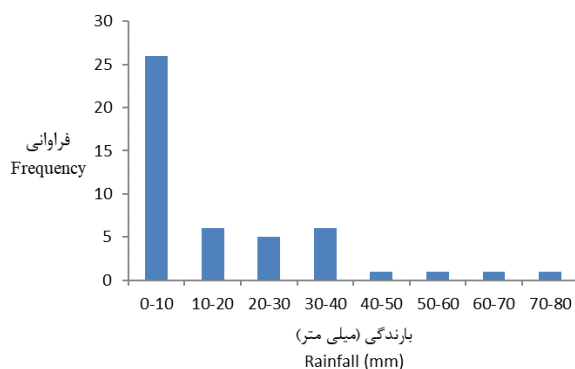


Figure 5. Observed frequency distribution of February rainfall at Zabol, 1969-2020

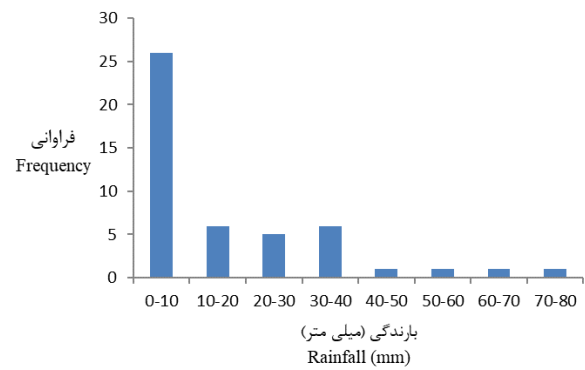


Figure 6. Observed frequency distribution of March rainfall at Zabol, 1969-2020

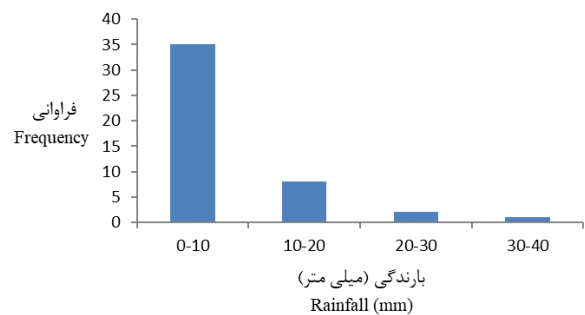


Figure 7. Observed frequency distribution of April rainfall at Zabol, 1969-2020

Table 3 compares the quantiles of monthly rainfall estimated by the exponential distribution with those estimated by the log-Pearson type III distribution, for return periods between 2 and 100 years, for each of the four wettest months. For January and February the two distributions produce broadly comparable quantiles across the full range of return periods, with the exponential distribution tending to estimate slightly higher rainfall depths at short and intermediate return periods. For March, the two distributions diverge more noticeably at long return periods, where the log-Pearson type III curve flattens while the exponential distribution, having a constant coefficient of variation of 1, continues to project comparatively higher quantiles; a similar, though less pronounced, pattern is seen for April. These differences illustrate the practical consequence of the exponential distribution's single-parameter form: it cannot independently adjust its skewness to match that of the sample, so that its performance is most reliable in months, such as January and February, whose sample coefficient of variation is closest to 1 (Table 1).

Table 3. Quantiles of Zabol January rainfall (mm) estimated by the log-Pearson type III and exponential distributions

Return period (yr)	Log-Pearson III (mm)	Exponential (mm)
2	9.3	13.3
5	25.9	30.8
10	43.1	44.1

20	64.5	57.4
25	72.6	61.7
50	100.7	74.9
100	134.2	88.2

Table 4. Quantiles of Zabol February rainfall (mm) estimated by the log-Pearson type III and exponential distributions

Return period (yr)	Log-Pearson III (mm)	Exponential (mm)
2	8.7	10.5
5	21.7	24.3
10	32.0	34.7
20	42.5	45.2
25	46.0	48.6
50	56.9	59.1
100	68.1	69.5

Table 5. Quantiles of Zabol March rainfall (mm) estimated by the log-Pearson type III and exponential distributions

Return period (yr)	Log-Pearson III (mm)	Exponential (mm)
2	11.8	10.5
5	25.5	24.3
10	33.5	34.8
20	39.8	45.3
25	41.5	48.7
50	45.9	59.2
100	49.2	69.7

Table 6. Quantiles of Zabol April rainfall (mm) estimated by the log-Pearson type III and exponential distributions

Return period (yr)	Log-Pearson III (mm)	Exponential (mm)
2	4.5	5.5
5	12.5	12.9
10	20.2	18.4
20	29.1	24.0
25	32.3	25.8
50	42.8	31.3
100	54.5	36.8

Overall, the exponential distribution reproduced the monthly rainfall quantiles of Zabol with an accuracy broadly comparable to that of the two-parameter gamma and log-Pearson type III distributions, despite requiring the estimation of only one parameter rather than two or three. This parsimony is particularly advantageous in hyper-arid regions, where rainfall records are often short or contain a large number of near-zero or zero values, so

that the higher-order sample moments needed to estimate the parameters of the gamma and log-Pearson type III distributions can be unstable. The results also confirm that the goodness of fit of the exponential distribution improves once months with a high proportion of near-zero rainfall are treated separately from the wetter months, consistent with the theoretical requirement that the coefficient of variation of the sample be reasonably close to 1.

4. Conclusions

This study evaluated the one-parameter exponential distribution for the frequency analysis of monthly rainfall at Zabol, a hyper-arid station in south-eastern Iran, based on a 52-year monthly rainfall record. The main conclusions are: (i) the exponential distribution, whose single parameter under the method of moments is simply the reciprocal of the mean, provides a reasonably good description of the frequency distribution of monthly rainfall at Zabol, particularly for the wetter months of January and February; (ii) the rainfall quantiles estimated by the exponential distribution are broadly comparable to those obtained from the more heavily parameterised two-parameter gamma and log-Pearson type III distributions, while requiring substantially less information to fit; and (iii) because the exponential distribution is a special case of both the gamma and the Weibull distributions, it offers a mathematically consistent and computationally simple starting point for the frequency analysis of rainfall in other hyper-arid regions with similarly short or sparse records. To the authors' knowledge, this is the first application of the exponential distribution to the frequency analysis of monthly rainfall in Iran, and the approach is offered as a practical complement, rather than a replacement, to conventional two- and three-parameter distributions in data-scarce hyper-arid settings.

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6. References

- Abolverdi, J., & Khalili, D. (2010). Development of regional rainfall annual maxima for southwestern Iran by L-moments. *Water Resources Management*, 24(11), 2501-2526.
- Alamatsaz, M. H., & Bidram, H. (2007). Some properties of the exponential distribution and its applications. *Journal of Mathematical Statistics*, 1(1), 1-8. (In Persian)
- Anonymous. (1969-2020). *Meteorological Yearbook of Zabol Synoptic Station*. Islamic Republic of Iran Meteorological Organization, Tehran.
- Brase, C. H., & Brase, C. P. (2012). *Understandable Statistics: Concepts and Methods* (10th ed.). Brooks/Cole, Boston.

- Eagleson, P. S. (1972). Dynamics of flood frequency. *Water Resources Research*, 8(4), 878-898.
- Gregory, S. (1969). Rainfall probability. In: *Water, Earth and Man* (R. J. Chorley, ed.), Methuen, London.
- Haghighat jou, P. (1989). *Frequency Analysis of Annual and Seasonal Precipitation in Sistan Region* (MSc thesis). University of Tehran, Tehran.
- Haghighat jou, P. (2002). Frequency analysis of monthly and annual precipitation of Zabol station. *Journal of Agricultural Sciences and Natural Resources*, 9(3), 41-48. (In Persian)
- Haghighat jou, P., Mirlatifi, S. M., & Sohrabi, T. (2009). Regional flood frequency analysis using L-moments for the arid and semi-arid regions of Iran. *Research Journal of Environmental Sciences*, 3(1), 62-70.
- Haghighat jou, P., Telvari, A. R., & Mirhashemi, S. H. (2013). Regional analysis of drought using Palmer Drought Severity Index (PDSI) in arid and semi-arid regions of Iran. *Theoretical and Applied Climatology*, 112(1-2), 193-200.
- Khalili, D. (1973). *Statistical Analysis of Precipitation Data of Iran* (Technical Report). Ministry of Water and Power, Tehran.
- Khalili, D. (1976). *Frequency Analysis of Hydrological Data*. University of Tehran Press, Tehran.
- Kite, G. W. (1978). *Frequency and Risk Analyses in Hydrology*. Water Resources Publications, Fort Collins, Colorado.
- Linsley, R. K., Kohler, M. A., & Paulhus, J. L. H. (1982). *Hydrology for Engineers* (3rd ed.). McGraw-Hill, New York.
- Modarres, R. (2006). Regional precipitation climates of Iran. *Journal of Hydrology*, 45, 13-27.
- Mooley, D. A., & Appa Rao, G. (1971). Distribution function for seasonal and annual rainfall over India. *Monthly Weather Review*, 99(10), 796-799.
- Neter, J., Wasserman, W., & Kutner, M. H. (1988). *Applied Linear Regression Models* (2nd ed.). Irwin, Homewood, Illinois.
- Rao, A. R., & Hamed, K. H. (2000). *Flood Frequency Analysis*. CRC Press, Boca Raton, Florida.
- Sen, Z., & Eljadid, A. G. (1999). Rainfall distribution function for Libya and rainfall prediction. *Hydrological Sciences Journal*, 44(5), 665-680.
- Sharma, T. C. (1996). Simulation of the Kenyan longest dry and wet spells and the largest rainfall using deseasonalized daily rainfall data. *Water Resources Management*, 10(2), 147-162.
- Singh, V. P. (1998). *Entropy-Based Parameter Estimation in Hydrology*. Kluwer Academic Publishers, Dordrecht.