

Flood Frequency Analysis Using the Fourier Series Method

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Abstract: Reliable estimates of flood quantiles are essential for the design of dams, spillways, flood-control structures and reservoir operating rules, yet the parametric probability distributions conventionally used for flood frequency analysis can fit annual peak-flow data poorly when the sample is small, multimodal or heavily skewed. This study compares ten parametric distributions (two- and three-parameter log-normal, two-parameter gamma, Pearson and log-Pearson type 3, Gumbel/extreme value type I, logistic, log-logistic, generalized logistic and generalized log-logistic) with four nonparametric approaches (normal, log-normal and triangular kernel functions, and the Fourier series method) for the frequency analysis of annual peak flows at six hydrometric stations in Khuzestan province, Iran, with record lengths of 43 to 47 years. Parameters of the parametric distributions were estimated by both the method of moments and maximum likelihood, and the smoothing parameter of the kernel functions was obtained by the rule of thumb, the Sheather-Jones and the least-squares cross-validation methods. The goodness of fit of all fourteen methods was assessed using the mean relative deviation (MRD) and the mean square relative deviation (MSRD) between observed and computed flood values. The log-Pearson type 3 distribution gave the best fit among the parametric distributions for five of the six stations, and the generalized log-logistic distribution gave the best fit for the remaining station; among the three kernel functions, the log-normal kernel performed best. However, the Fourier series method outperformed all ten parametric distributions and all three kernel functions at every station, with substantially lower MRD and MSRD values. Because the Fourier series method requires no assumption about the underlying probability distribution and no choice among competing parameter-estimation procedures, it is proposed as a robust, easily applied alternative for flood frequency analysis in the study region and in other basins with similarly short or irregularly shaped flood records.

Keywords: Parametric and nonparametric methods, Probability distributions, Annual floods, Fourier series, Kernel function

1. Introduction

Reliable estimates of the magnitude and return period of annual peak flows are required to size dams, spillways and flood-control structures, to operate reservoirs for flood control, water supply and power production, and to support watershed management, drought monitoring and climate-change studies. This information is conventionally obtained by fitting a statistical probability distribution, most often the two- or three-parameter log-normal, two-parameter gamma, Pearson or log-Pearson type III, or the Gumbel/extreme value type I distribution, to a series of observed annual peak flows. The U.S. Federal Water Resources Council recommended the log-Pearson type III distribution as the standard flood-frequency model for U.S. government agencies (Benson, 1968), and the Gumbel distribution has likewise been widely adopted, including in the United Kingdom (NERC, 1975; Linsley et al., 1982). These parametric distributions have been applied

successfully in many settings, but they can fit observed data poorly, particularly at the extremes and for multimodal samples, and their use raises the practical difficulty of choosing both the most appropriate distribution and the most appropriate parameter-estimation procedure for a given record (Ahmed et al., 1988; Al Yasin, 2000).

Nonparametric methods offer an alternative that avoids assuming a fixed distributional shape in advance. Kronmal and Tarter (1968) proposed the Fourier series method as a nonparametric approach for estimating probability density and cumulative distribution functions, and orthogonal functions have since been used more broadly to construct density estimators (Devroye and Györfi, 1985). Kernel density and distribution estimators, and the Fourier series method, have subsequently been applied to a range of hydrological problems (Lall et al., 1993). Wu and Woo (1989) compared the Fourier series method with four

parametric and two nonparametric methods for eight Canadian rivers and found that the nonparametric methods performed better overall; Moon et al. (1993) compared several tail-probability and variable-kernel estimators for annual maximum floods and found the variable-kernel estimator useful; and Adamowski (2000) applied a Gaussian kernel function to regional annual-maximum and partial-duration flood series, concluding that conventional parametric approaches can be deficient when flood data are multimodal. Faucher et al. (2002) compared parametric and nonparametric quantile estimators on simulated samples and found that nonparametric kernel methods performed comparably to parametric distributions, with little difference among the biweight, normal, Epanechnikov, extreme-value, rectangular and Cauchy kernels.

Several studies specific to Iran have applied the Fourier series method to hydro-meteorological frequency analysis. Behnia and Haghighat jou (2007) applied the Fourier series method to the annual flood probabilities of the Great Karoun river and found it outperformed seven parametric methods. Haghighat jou et al. (2009) compared seven parametric distributions with the Fourier series method for the frequency analysis of annual precipitation in Iran and likewise found the nonparametric estimator fitted the observed data better. Haghighat jou et al. (2013) fitted annual precipitation from five long-record Iranian stations using rectangular, triangular and Gaussian kernel functions, and found that the kernel functions outperformed the parametric distributions while the Fourier series method outperformed the kernel functions in turn. Haghighat jou and Mirhashemi (2023) reached a similar conclusion for the frequency analysis of extreme daily rainfall at fourteen arid-zone stations. Karmakar and Simonovic (2008) used kernel density estimation and an orthonormal cosine series to derive the marginal distributions of flood peak, volume and duration for a bivariate flood-frequency analysis, and found the orthonormal-series approach more appropriate than kernel estimation for this purpose.

Despite this body of evidence, the Fourier series method remains far less widely used in practice than conventional parametric distributions, and a systematic comparison of the method against the full range of parametric and kernel-based alternatives for flood frequency analysis in south-western Iran has not previously been reported. The main purpose of this paper is therefore to determine the best-performing method for flood frequency analysis among ten parametric distributions and four nonparametric approaches, using annual peak-flow records from six hydrometric stations in Khuzestan province.

2. Materials and Methods

2.1. Study Area and Data

Annual peak-flow series from six hydrometric stations across Khuzestan province, south-western

Iran, were selected for frequency analysis. The stations have natural, unregulated flow regimes and record lengths of 43 to 47 years. The data were fitted to ten parametric probability distributions — two- and three-parameter log-normal, two-parameter gamma, Pearson and log-Pearson type 3, Gumbel/extreme value type I, logistic, log-logistic, generalized logistic and generalized log-logistic — and to four nonparametric approaches: normal, log-normal and triangular kernel functions, and the Fourier series method. Table 1 summarises the mean, coefficient of variation and coefficient of skewness of the annual flood series at each station. Parameters of the parametric distributions and the mathematical formulation of the kernel and Fourier series methods follow standard references (Haan, 1977; Kite, 1988; Rao and Hamed, 2000; Wu and Woo, 1989; Behnia and Haghighat jou, 2007; Haghighat jou et al., 2009, 2013) and are not repeated here.

Table 1. Statistics of the annual flood series for the six rivers/stations

River	Station	Record length (yr)	Mean (m ³ /s)	CV	C _{sk}
Dez	Taleh zang	47	2985.8	0.60	1.09
Maroun	Idenak	45	1218.7	0.89	2.16
Karoun	Pole shalou	47	2035.8	0.58	1.27
Karkheh	Paye pol	43	1758.4	0.54	1.15
Kheirabad	Kheirabad	46	775.2	0.79	1.09
Zohreh	Gachsaran	46	1026.2	0.84	1.42

CV: coefficient of variation; C_{sk}: coefficient of skewness.

2.2. Goodness-of-Fit Criteria

The goodness of fit of the parametric and nonparametric methods was measured by the mean relative deviation (MRD) and the mean square relative deviation (MSRD) between observed and estimated annual flood values:

$$\text{MRD} = (1/n) \sum (|x_i - \hat{x}_i| / x_i) \times 100 \quad (1)$$

$$\text{MSRD} = (1/n) \sum [(x_i - \hat{x}_i) / x_i \times 100]^2 \quad (2)$$

where x and $\hat{x}$ are, respectively, the observed annual flood and its value estimated by a given method, and n is the sample size. Observed and estimated distribution functions were also compared graphically for each station.

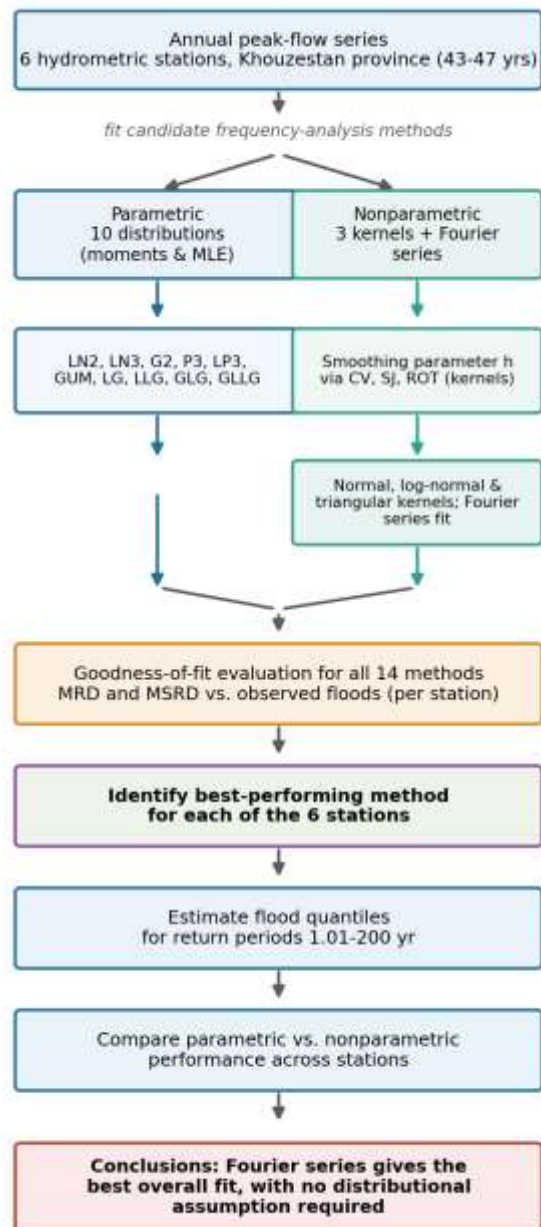


Figure 1. Methodological flowchart of the comparative flood frequency analysis

2.3. Parameter Estimation and Selection of the Smoothing Parameter

For the ten parametric distributions, parameters were estimated by both the method of moments and the method of maximum likelihood (Haan, 1977; Kite, 1988; Rao and Hamed, 2000). For the three nonparametric kernel functions, the smoothing parameter h was calculated by three independent methods—the rule of thumb (ROT), the Sheather-Jones method (SJ) and least-squares cross-validation (LSCV)—summarised in Table 2. The values obtained by ROT differ markedly from those obtained by SJ and LSCV, which are themselves reasonably close to each other; the LSCV value of h was therefore adopted for the kernel-

based analysis reported below, following Faucher (1999), Faucher et al. (2001) and Haghighatjou (2009). Once h was fixed, an estimated flood value was obtained for every observation under each kernel function, and Eqs (1) and (2) were then applied to all fourteen methods to identify the best-fitting parametric distribution and the best-fitting nonparametric approach at each station.

Table 2. Values of the smoothing parameter, h , for the six stations by the rule-of-thumb (ROT), Sheather-Jones (SJ) and least-squares cross-validation (LSCV) methods

River	Station	ROT	SJ	LSCV
Dez	Taleh zang	882.03	642.32	693.61
Maroun	Idenak	558.60	380.52	348.23
Karoun	Pole shalou	594.54	410.30	326.45
Karkheh	Paye pol	476.28	335.27	387.43
Kheirabad	Kheirabad	302.32	193.18	132.66
Zohreh	Gachsaran	426.60	208.58	189.44

3. Results and Discussion

Tables 3 and 4 report the MRD and MSRD values obtained for all ten parametric distributions and four nonparametric methods at each of the six stations (the value for the Pearson type 3 distribution at Pole shalou station is omitted, marked —, because the fitted distribution did not converge for this station). Comparing these values, the log-Pearson type 3 distribution gives the best fit among the parametric distributions for every station except Paye pol, where the generalized log-logistic distribution performs best. Considering all fourteen methods together, the Fourier series method fits the observed flood data at every station markedly better than any of the parametric distributions or kernel functions, with MRD values of only 1.05-2.33% and MSRD values of only 1.82-33.04%, compared with values ranging up to several tens of percent for the parametric distributions and up to several thousand for the least well-fitting distributions. Among the three kernel functions, the log-normal kernel gives the closest fit at every station, a result that differs from Faucher et al. (2002), who found little difference among several kernel functions in their simulation study.

Table 3. Values of MRD (%) for the ten parametric distributions and four nonparametric methods, by river/station

Method	Dez	Maroun	Karoun	Karkheh	Kheirabad	Zohreh
LN2	8.79	10.42	7.91	11.49	30.16	17.75
LN3	7.69	16.10	8.46	5.94	35.31	30.35
G2	5.23	12.88	7.80	6.86	9.60	12.51
P3	4.97	15.49	—	9.83	21.75	14.71
LP3	4.75	6.23	6.61	6.50	7.40	7.46
GUM	7.55	41.89	9.41	5.70	34.86	38.90
LG	25.24	35.66	54.68	18.98	92.25	82.77
LLG	7.07	14.43	22.45	10.55	15.77	10.87
GLG	8.22	13.56	8.23	7.76	29.00	19.25

GLLG	6.02	8.41	14.43	5.93	10.00	9.52
NK	10.40	19.23	6.38	9.50	16.69	13.91
LNK	8.35	13.52	7.23	3.91	11.98	11.80
TK	18.62	27.56	5.72	3.33	4.76	24.30
FS	1.29	2.33	1.94	1.05	1.93	1.71

Table 4. Values of MSRD (%) for the ten parametric distributions and four nonparametric methods, by river/station

Method	Dez	Maroun	Karoun	Karkheh	Kheirabad	Zohreh
LN2	188.97	243.98	86.61	626.09	2944.68	630.50
LN3	213.24	1570.45	162.44	83.08	116.35	5661.48
G2	55.20	522.88	128.10	125.41	164.43	335.68
P3	45.32	1901.43	—	454.03	3343.84	590.10
LP3	42.66	73.90	63.36	73.15	95.84	87.42
GUM	213.39	131.91	232.99	73.43	116.41	107.50
LG	3717.44	24562.32	70861.87	3326.33	103055.80	67866.10
LLG	106.93	3248.59	5348.94	248.63	737.79	213.88
GLG	204.42	780.07	134.59	306.35	7499.49	1675.10
GLLG	66.88	118.89	2338.78	59.07	196.68	190.06
NK	532.84	1854.17	135.64	499.23	2834.30	1379.20
LNK	104.63	185.73	92.07	23.10	221.44	248.52
TK	353.41	407.58	520.93	41.90	218.39	453.78
FS	4.01	12.35	33.04	1.82	13.44	7.56

Method abbreviations: LN2/LN3, two-/three-parameter log-normal; G2, two-parameter gamma; P3, Pearson type 3; LP3, log-Pearson type 3; GUM, Gumbel; LG, logistic; LLG, log-logistic; GLG, generalized logistic; GLLG, generalized log-logistic; NK/LNK/TK, normal/log-normal/triangular kernel; FS, Fourier series.

Figures 2-4 illustrate the fit of the observed and calculated annual floods for the Karkheh river at Paye pol station — respectively for the generalized log-logistic distribution (the best-fitting parametric distribution at this station), the Fourier series method, and the log-normal kernel function (the best-fitting kernel function) — and the relative goodness of fit shown is representative of the other five stations.

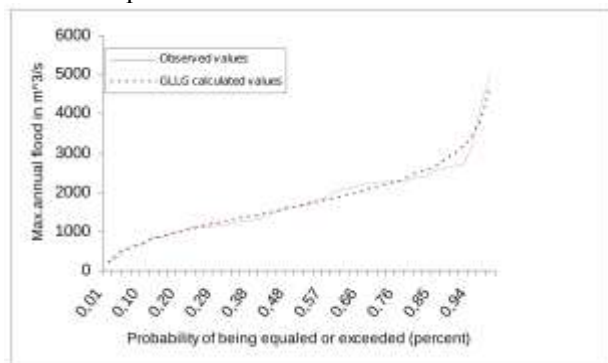


Figure 2. Observed and calculated values of annual floods by the generalized log-logistic (GLLG) distribution for the Karkheh river at Paye pol station

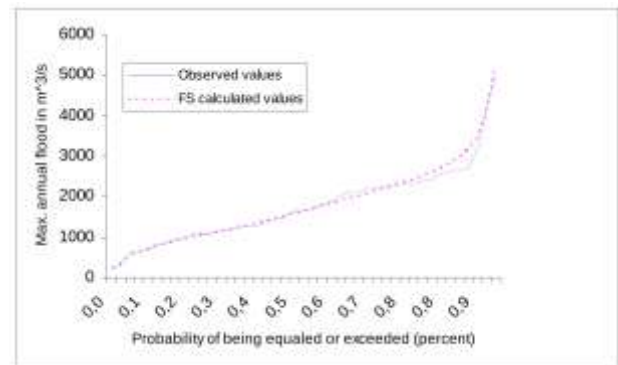


Figure 3. Observed and calculated values of annual floods by the Fourier series (FS) method for the Karkheh river at Paye pol station

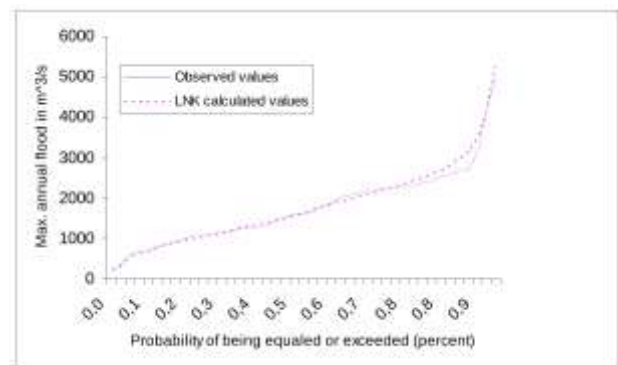


Figure 4. Observed and calculated values of annual floods by the log-normal kernel (LNK) function for the Karkheh river at Paye pol station

Table 5 lists the annual flood quantiles estimated for return periods from 1.0101 to 200 years by the log-normal kernel function, the Fourier series method, and the best-fitting parametric distribution (LP3 or GLLG, as appropriate) for the Kheirabad, Paye pol and Taleh zang stations, chosen to illustrate the range of station sizes. At long return periods (50, 100 and 200 years) the quantiles estimated by the Fourier series method are close to those of the best-fitting parametric distribution, while at short and intermediate return periods the three methods diverge more visibly, particularly at Taleh zang, the largest of the six stations, where the log-normal kernel and the Fourier series method diverge from log-Pearson type 3 in opposite directions at the 100-year return period.

Table 5. Annual flood quantiles for the Kheirabad river at Kheirabad station, estimated by the log-normal kernel (LNK), Fourier series (FS) and log-Pearson type 3 (LP3) methods

Return period (yr)	Prob. of exc.	LNK (m³/s)	FS (m³/s)	LP3 (m³/s)
1.0101	0.990	29.5	32.5	32.4
1.0256	0.975	45.5	48.8	57.4
1.0526	0.950	73.2	79.7	90.5
1.1111	0.900	125.5	161.1	147.3
1.2500	0.800	217.2	253.0	252.2
2	0.500	587.0	663.2	607.3
5	0.200	1387.1	1233.8	1222.6
10	0.100	2043.7	1627.4	1652.3

20	0.050	2750.1	2118.3	2054.9
25	0.040	2988.5	2275.0	2178.8
50	0.020	3763.9	2435.6	2546.4
100	0.010	4594.4	2462.0	2887.8
200	0.005	5481.9	2474.4	3203.8

Table 6. Annual flood quantiles for the Karkheh river at Paye pol station, estimated by the log-normal kernel (LNK), Fourier series (FS) and generalized log-logistic (GLLG) methods

Return period (yr)	Prob. of exc.	LNK (m ³ /s)	FS (m ³ /s)	GLLG (m ³ /s)
1.0101	0.990	214.9	222.9	179.2
1.0256	0.975	268.9	273.9	323.1
1.0526	0.950	461.3	553.3	481.2
1.1111	0.900	683.4	683.9	695.0
1.2500	0.800	947.1	1001.3	988.2
2	0.500	1630.4	1627.2	1636.8
5	0.200	2489.6	2351.8	2450.4
10	0.100	2983.3	2661.0	2985.3
20	0.050	3649.5	3215.4	3504.7
25	0.040	3931.4	4130.1	3670.9
50	0.020	4743.0	4934.8	4185.3
100	0.010	5363.8	4985.4	4696.7
200	0.005	5879.4	5010.7	5202.2

Table 7. Annual flood quantiles for the Dez river at Taleh zang station, estimated by the log-normal kernel (LNK), Fourier series (FS) and log-Pearson type 3 (LP3) methods

Return period (yr)	Prob. of exc.	LNK (m ³ /s)	FS (m ³ /s)	LP3 (m ³ /s)
1.0101	0.990	405.0	536.1	465.6
1.0256	0.975	525.7	586.8	632.3
1.0526	0.950	686.2	667.0	813.7
1.1111	0.900	952.2	1131.9	1075.1
1.2500	0.800	1357.8	1517.1	1481.0
2	0.500	2543.7	2614.3	2599.3
5	0.200	4702.5	4435.5	4283.7
10	0.100	6310.5	5415.2	5431.3
20	0.050	7928.0	6560.2	6530.0
25	0.040	8452.1	6741.5	6876.5
50	0.020	10096.0	8476.8	7935.6
100	0.010	11765.0	8553.6	8973.2
200	0.005	13462.4	8591.6	10284.3

Unlike the parametric distributions, the Fourier series method imposes no assumption about the shape of the underlying probability distribution, which allows it to accommodate the irregular, occasionally multimodal, empirical distributions typical of short flood records. It also removes the need to select among competing

parameter-estimation procedures — method of moments, maximum likelihood or probability-weighted moments — since its coefficients are obtained directly from the empirical distribution. These two properties, combined with its consistently superior MRD and MSRD performance at all six stations, are the practical advantages of the Fourier series method over the parametric approaches evaluated in this study, and are consistent with the findings of Wu and Woo (1989) for Canadian rivers, Behnia and Haghighat jou (2007) for the Great Karoun river, and Haghighat jou and Mirhashemi (2023) for extreme daily rainfall in Iran. Karmakar and Simonovic (2008), using a 70-year flood peak, volume and duration record for the Red River, similarly found an orthonormal cosine-series approach more appropriate than kernel estimation for determining the marginal distributions of flood characteristics over their full range of possible values.

4. Conclusions

Annual flood series from six hydrometric stations in Khuzestan province, Iran, were fitted to ten parametric probability distributions and four nonparametric approaches — the Fourier series method and normal, log-normal and triangular kernel functions — to identify the best-performing method for flood frequency analysis at each station. The main conclusions are: (i) among the parametric distributions, the log-Pearson type 3 distribution gave the best fit for five of the six stations, and the generalized log-logistic distribution gave the best fit for the remaining station (Paye pol); (ii) among the three kernel functions, the log-normal kernel gave the closest fit at every station; (iii) the Fourier series method fitted the observed annual flood data at every station markedly better than any of the ten parametric distributions or three kernel functions, as measured by both MRD and MSRD; and (iv) because the Fourier series method requires neither a distributional assumption nor a choice among competing parameter-estimation procedures, it removes a recurring source of uncertainty and disagreement in conventional flood frequency analysis. These findings, consistent with previous applications of the Fourier series method to Iranian rivers and rainfall series, support its use as a robust, easily applied alternative for flood frequency analysis in the study region and in other basins with similarly short or irregularly shaped flood records.

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6. References

- Ahmed, M. I., Sinclair, C. D., & Werritty, A. (1988). Log-logistic flood frequency analysis. *Journal of Hydrology*, 98(3-4), 205-224.
- Adamowski, K. (2000). Regional analysis of annual maximum and partial duration flood data by

- nonparametric and L-moment methods. *Journal of Hydrology*, 229(3-4), 219-231.
- Al Yasin, A. (2000). *Application of River Engineering in Dez and Karoun Rivers*. Ministry of Power of I. R. of Iran, Issue No. 33, National Committee on Large Dams of Iran. (In Persian)
- Anonymous. (1995). *HYFA (Hydrological Frequency Analysis) Software*.
- Behnia, A. K., & Haghighat jou, P. (2007). Estimating Karoun annual flood probabilities using the Fourier series method. *Proceedings of the 7th International River Engineering Conference*, Feb. 13-15, Shahid Chamran University, Ahvaz, Iran, 1-7. (In Persian)
- Benson, M. A. (1968). Uniform flood-frequency estimating methods for federal agencies. *Water Resources Research*, 4(5), 891-908.
- Devroye, L., & Györfi, L. (1985). *Nonparametric Density Estimation: The L_1 View* (1st ed.). John Wiley and Sons, New York.
- Faucher, D. (1999). *Estimation Nonparamétrique des Quantiles de Crue par la Méthode des Noyaux* (MSc thesis). INRS-Eau, Quebec.
- Faucher, D., Rasmussen, P. F., & Bobee, B. (2001). A distribution function based bandwidth selection method for kernel quantile estimation. *Journal of Hydrology*, 250(1-4), 1-11.
- Faucher, D., Rasmussen, P. F., & Bobee, B. (2002). Nonparametric estimation of quantiles by the kernel method. *Journal of Water Science*, 15, 515-541. (In French)
- Haan, C. T. (1977). *Statistical Methods in Hydrology* (1st ed.). Iowa State University Press, Ames, Iowa.
- Haghighat jou, P. (2009). *A Comparative Evaluation of Parametric and Nonparametric Statistics in Precipitation Frequency Analysis* (PhD thesis). Shahid Chamran University, Ahvaz, Iran. (In Persian)
- Haghighat jou, P., Akhoond-Ali, A. M., Behnia, A. K., & Chinipardaz, R. (2009). A comparison of parametric and nonparametric density functions for estimating annual precipitation in Iran. *Research Journal of Environmental Sciences*, 3(1), 62-70.
- Haghighat jou, P., Akhoond-Ali, A. M., & Nazemosadat, M. J. (2013). Nonparametric-kernel estimation of annual precipitation over Iran. *Theoretical and Applied Climatology*, 112(1-2), 193-200.
- Haghighat jou, P., & Mirhashemi, S. H. (2023). Frequency analysis of extreme daily rainfall over an arid zone of Iran using the Fourier series method. *Applied Water Science*, 13, 16.
- Karmakar, S., & Simonovic, S. P. (2008). Bivariate flood frequency analysis using copula with parametric and nonparametric marginals. *4th International Symposium on Flood Defense: Managing Flood Risk, Reliability and Vulnerability*, Toronto, Canada, May 6-8.
- Kite, G. W. (1988). *Frequency and Risk Analyses in Hydrology* (4th ed.). Water Resources Publications, Fort Collins, Colorado.
- Kronmal, R., & Tarter, M. (1968). The estimation of probability densities and cumulatives by Fourier series methods. *Journal of the American Statistical Association*, 63(323), 925-952.
- Lall, U., Moon, Y. I., & Bosworth, K. (1993). Kernel flood frequency estimators: Bandwidth selection and kernel-shape sensitivity. *Water Resources Research*, 29(4), 1003-1015.
- Linsley, R. K., Kohler, M. A., & Paulhus, J. L. H. (1982). *Hydrology for Engineers* (3rd ed.). McGraw-Hill, New York.
- Moon, Y. I., Lall, U., & Bosworth, K. (1993). A comparison of tail probability estimators for flood frequency analysis. *Journal of Hydrology*, 151(2-4), 343-363.
- NERC. (1975). *Flood Studies Report* (5 volumes). Natural Environment Research Council, HMSO, London.
- Rao, A. R., & Hamed, K. H. (2000). *Flood Frequency Analysis* (1st ed.). CRC Press, Boca Raton, Florida.
- Wu, K., & Woo, M. K. (1989). Estimating annual flood probabilities using the Fourier series method. *Water Resources Bulletin*, 25(4), 743-750.